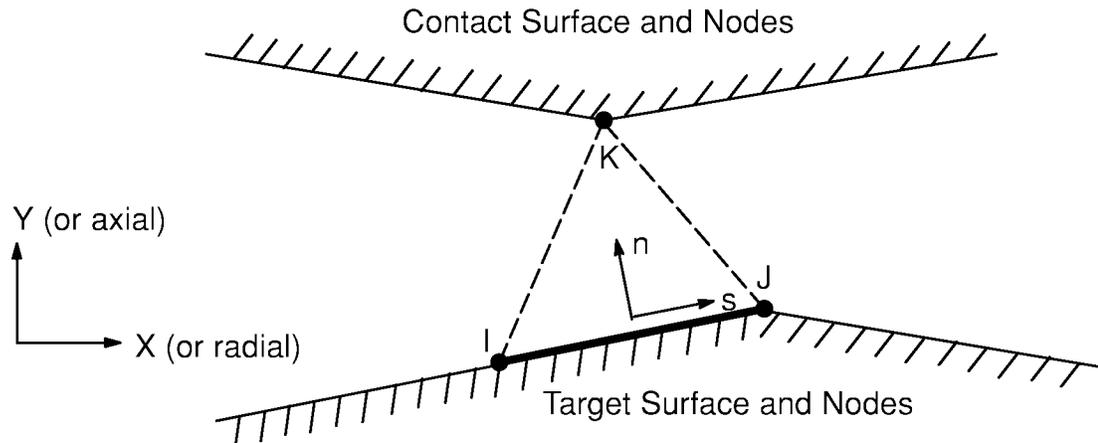


14.48 CONTAC48 — 2-D Point-to-Surface Contact



14.48.1 Introduction

CONTAC48 is a 3-node element that is intended for general contact analysis. In a general contact analysis, the area of contact between two (or more) bodies is generally not known in advance. In addition, the finite element models of the contacting bodies are generated in such a way that precise node-to-node contact is neither achievable nor desirable when contact is established. This type of contact situation precludes the use of node-to-node contact elements such as CONTAC12. CONTAC48 has the capability to represent general contact of models that are generated with arbitrary meshes. In other words, its use is not limited to known contact or node-to-node configurations.

CONTAC48 is applicable to 2-D geometries: plane strain, plane stress, or axisymmetry. It may be applied to contact of solid bodies or beams, to static or dynamic analyses, to problems with or without friction, and to flexible-to-flexible or rigid-to-flexible body contact. The combined mechanisms of structural contact and thermal contact conductance can also be represented by CONTAC48.

14.48.2 Contact Kinematics

Contact kinematics is concerned with the precise tracking of contact nodes and surfaces in order to define clear and unambiguous contact conditions. The primary aim

is to delineate between open (i.e., not in contact) and closed (in contact) contact situation. This task is accomplished by various algorithms embedded in CONTACT48.

Contact and target definition – With reference to the introductory figure, two potential contact surfaces are referred to as either the “target surface” or the “contact surfaces”. The “target surface” is represented by “target nodes” I and J, and the “contact surface” is represented by the “contact node” K. It is usually the case that many CONTACT48 elements will be needed to fully represent a realistic contact problem. (To that end the **GCGEN** command of the PREP7 routine can be used to generate CONTACT48 elements.)

Pinball algorithm – In simple terms, contact occurs whenever the contact node penetrates the target surface. The first step in the determination of contact penetration is to make a distinction between near-field and far-field contact. Figure 14.48–1 shows several positions of a contact node with respect to a circle centered on the target surface (nodes I and J). This circle is referred to as a “pinball”. When a contact node is outside the pinball an “open” contact condition is assumed, irrespective of whether or not the contact node K is above or below the target. Penetration can only occur if the contact node is inside the pinball. The radius of the pinball defaults to be 50% greater than the distance between the target nodes and can be overridden by real constant PINB.

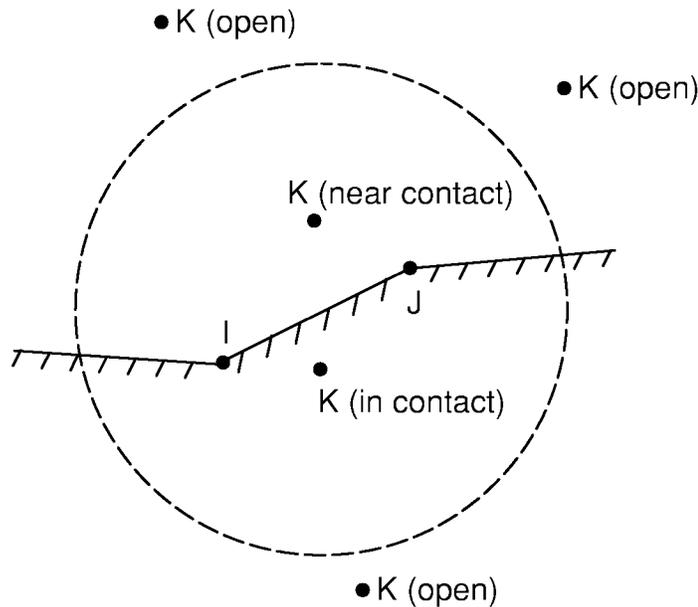


Figure 14.48–1 Definition of Near-Field and Far-Field Contact

Pseudo element algorithm – The next step in the determination of contact is to associate a single target to each contact node. Figure 14.48–2 shows the situation that occurs when one contact node is near (i.e., inside the pinball) several element targets. In particular it is seen that contact penetration has occurred and involves either the element defined by nodes I', J', and K or the element defined by nodes I'', J'', and K. If

a clear distinction is not made it is possible that contact “voids” or “overlaps” can appear (see Figure 14.48–3). These voids and overlaps are unavoidable and are due, in the main, to piecewise discretization of surfaces that are actually curved. To remove this potential difficulty, solid “pseudo elements” are formed that surround each target. These solid elements are temporarily formed each equilibrium iteration and provide a continuous mapping for each contact node that is in or nearly in contact with a target. The kinematic information that is needed to build these pseudo elements is stored in a global contact data base that is updated each equilibrium iteration.

As can be seen in Figure 14.48–4, the pseudo element mapping indicates that node K is uniquely assigned to the target of the element defined by nodes I', J', and K and not to the element defined by I'', J'', and K.

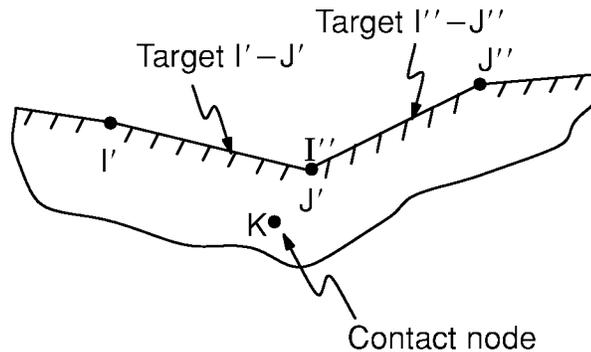


Figure 14.48–2 Contact Node with Two Potential Targets for Contact

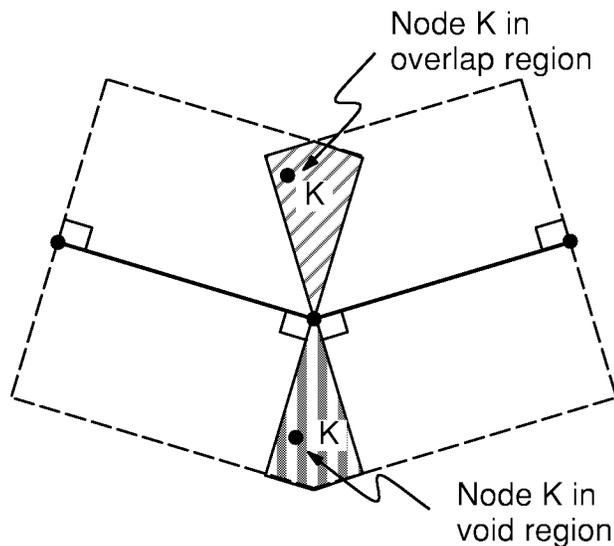


Figure 14.48–3 Potential Voids and Overlaps at Contact Intersection

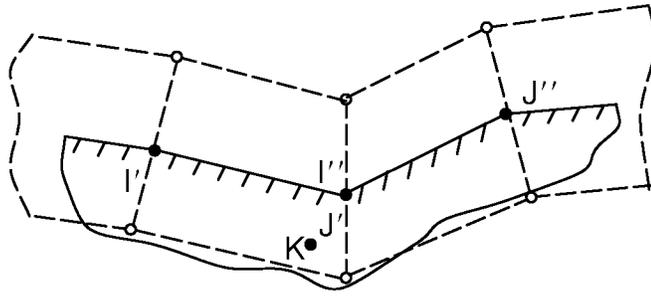


Figure 14.48-4 Pseudo Element

Contact gap and projection – The pinball and pseudo–element algorithms provide a one–to–one mapping between a contact node and a target. The final kinematic step is to calculate the amount of the open gap or the gap penetration of the contact node on the target plane, along with the point of projection of the contact node. In Figure 14.48-5 an element coordinate system is indicated. It is useful to first define the unit normal and unit tangent vector to the target plane.

$$\{n\} = \{v\} \times \{s\} \quad (14.48-1)$$

$$\{s\} = (\{X_J\} - \{X_I\}) / L \quad (14.48-2)$$

where:

- $\{v\}$ = unit normal vector to the global x–y plane
- $\{X_I\}$ = position vector of node I
- $\{X_J\}$ = position vector of node J
- L = length of target segment = $\|\{X_J\} - \{X_I\}\|$

The gap (g) and projection point (s^*), defined in local s–n coordinates, are

$$g = (\{X_K\} - \{X_I\})^T \{n\} \quad (14.48-3)$$

$$s^* = -1 + 2 [(\{X_K\} - \{X_I\})^T \{s\}] / L \quad (14.48-4)$$

where: $\{X_K\}$ = position vector of node K

Contact penetration is assumed to occur if the value of g is found to be negative. It is important to mention that the target surface is internally expanded if input quantity TOLS on **R** command is specified, thereby increasing the chances that contact will occur. A positive gap value indicates an open contact condition. These various conditions of contact are referred to in CONTACT48's output as "status". These are

STAT (or OLDST) = 4 (open and outside the pinball)

STAT (or OLDST) = 3 (open and inside the pinball)

STAT (or OLDST) < 3 (contact has occurred) (STAT=1 or 2 are described below)

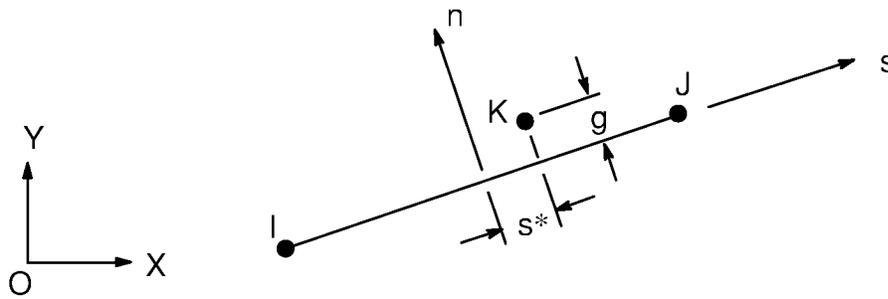


Figure 14.48-5 Location of Contact Node in Local Contact Coordinates

14.48.3 Contact Forces

As explained above contact is indicated when the contact node K penetrates the target surface defined by target nodes I and J. This penetration is represented by the magnitude of the gap (g) and is a violation of compatibility. In order to satisfy contact compatibility, forces are developed in a direction normal (n -direction) to the target that will tend to reduce the penetration to an acceptable numerical level. In addition to compatibility forces, friction forces are developed in a direction that is tangent (s -direction) to the target plane.

Normal forces – Two methods of satisfying contact compatibility are available for CONTAC48: a penalty method (KEYOPT(2)=0) or a combined penalty plus Lagrange multiplier method (KEYOPT(2)=1). The penalty method approximately enforces compatibility by means of a contact stiffness (i.e., the penalty parameter). The combined approach satisfies compatibility to a user-defined precision by the generation of additional contact forces that are referred to as Lagrange forces.

For the penalty method,

$$f_n = \begin{cases} K_n g & \text{if } g \leq 0 \\ 0 & \text{if } g > 0 \end{cases} \quad (14.48-5)$$

where: K_n = contact stiffness (input quantity KN on **R** command)

For the combined method, the Lagrange multiplier component of force is computed locally (for each element) and iteratively. It is expressed as

$$f_n = \min (0, K_n g + \lambda_{i+1}) \quad (14.48-6)$$

where:

$$\lambda_{i+1} = \begin{cases} \lambda_i + \alpha K_n g & \text{if } |g| \geq \varepsilon \\ \lambda_i & \text{if } |g| < \varepsilon \end{cases} \quad (14.48-7)$$

ε = user-defined compatibility tolerance (input quantity TOLN on **R** command)

α = an internally computed factor ($\alpha < 1$)

Tangential forces – Tangential forces are due to friction that arises as the contact node meets and moves along the target. CONTAC48 allows three friction models: frictionless (KEYOPT(3)=0), elastic Coulomb friction (KEYOPT(3)=1), and rigid Coulomb friction (KEYOPT(3)=2). The Coulomb friction representation requires the specification of the coefficient of sliding friction (μ), which is supplied by means of material property MU on the **MP** command.

For the frictionless case, the tangential force is merely

$$f_s = 0 \quad (14.48-8)$$

For elastic Coulomb friction it is necessary to calculate the tangential displacement of the contact node relative to the target. The total tangential motion is

$$u_s = \frac{1}{2} (s^* - s_0^*) L \quad (14.48-9)$$

where s_0^* is the contact position at the previous converged solution or time point.

The deformation is decomposed into elastic (or sticking) and sliding (inelastic) components:

$$u_s = u_s^e + u_s^s \quad (14.48-10)$$

where:

$$\begin{aligned} u_s^e &= \text{elastic tangential deformation} \\ u_s^s &= \text{sliding (inelastic) tangential deformation} \end{aligned}$$

The tangential force is

$$f_s = \begin{cases} K_t u_s^e < F \bar{f}_s & \text{if sticking (STAT = 1)} \\ \bar{f}_s & \text{if sliding (STAT = 2)} \end{cases} \quad (14.48-11)$$

where:

$$\begin{aligned} K_t &= \text{sticking stiffness (input quantity KT on } \mathbf{R} \text{ command)} \\ \bar{f}_s &= \text{sticking force limit of the Coulomb friction model} \\ F &= \text{static/dynamic friction factor (input quantity FACT on } \mathbf{R} \text{ command)} \end{aligned}$$

The limiting sticking force is

$$\bar{f}_s = -\mu f_n \quad (14.48-12)$$

where: μ = coefficient of sliding friction
 $= \tan \phi$ (see Figure 14.48-6)

Equations (14.48-5) to (14.48-12) are merely a summary of contact forces and displacements. The actual computation that is performed uses a technique that is similar to that of non-associative theory of plasticity (see Section 4.1). In each substep that sliding friction occurs, an elastic predictor is computed in contact traction space (Figure 14.48-6). The predictor is modified with a radial return mapping function, providing both a small elastic deformation along with sliding response as developed by Giannakopoulos (135).

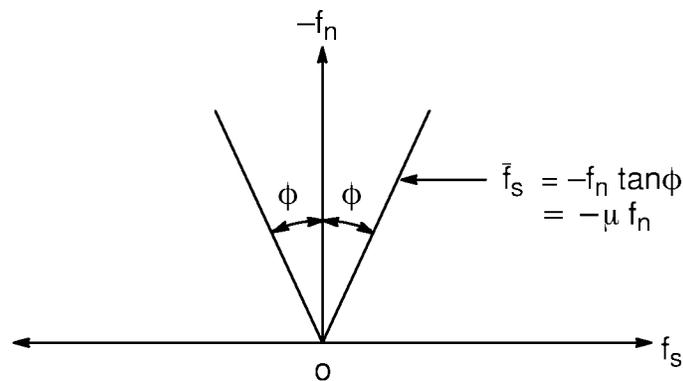


Figure 14.48-6 Contact Friction Space for Coulomb Friction

For the elastic Coulomb model, initial contact is always treated as elastic sticking (STAT=1), but with the tangential force set to zero ($f_s = 0$). In other words, the goal of initial contact is the determination of the penetration (g) and the contact point (s^*), irrespective of friction forces. All subsequent substeps will allow friction to develop according to equation (14.48-11).

Elastic contact tangential deformations are ignored in the rigid Coulomb friction model. The contact node K (if penetrated) is always assumed to be sliding on the target (STAT=2), where the tangential force is

$$f_s = \frac{u_s}{|u_s|} \bar{f}_s \quad (14.48-13)$$

Contact force transition – A special situation arises when a contact node moves from one target to another. When this occurs, the contact history is passed from the target that was in contact to the target that is currently subjected to contact. In so doing, the path-dependence of friction is maintained and, for some problems, convergence

behavior is seen to improve. The transition makes use of a contact data base that contain contact conditions and forces for all contact nodes in actual contact.

14.48.4 Stiffness Matrix and Load Vector

It is convenient to define two interpolation vectors in terms of the local s -direction coordinate. These interpolation vectors are evaluated at the point of projection ($s = s^*$) of the contact node K to the target plane (see Figure 14.48-5), where s^* is dimensionless and ranges from -1 to $+1$.

$$\{N_n\} = \left[0 \quad -\frac{1}{2}(1 - s^*) \quad 0 \quad -\frac{1}{2}(1 + s^*) \quad 0 \quad 1 \right]^T \quad (14.48-14)$$

$$\{N_s\} = \left[-\frac{1}{2}(1 - s^*) \quad 0 \quad -\frac{1}{2}(1 + s^*) \quad 0 \quad 1 \quad 0 \right]^T \quad (14.48-15)$$

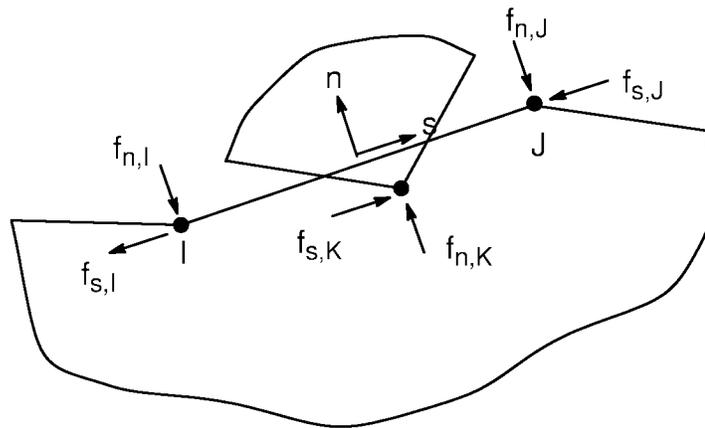


Figure 14.48-7 Nodal Contact Forces

Figure 14.48-7 shows all nodal contact forces. In the normal direction, the force applied to the contact node is defined in equation (14.48-6) and is balanced by opposite force applied to the target nodes; that is,

$$f_{n,K} = f_{n,I} + f_{n,J} = f_n \quad (14.48-16)$$

Similarly, in the tangential direction,

$$f_{s,K} = f_{s,I} + f_{s,J} = f_s \quad (14.48-17)$$

where f_s is defined by either equation (14.48-8), (14.48-11), and (14.48-13).

Using the interpolation vectors above, the element load vector (i.e., the Newton–Raphson restoring forces) is

$$\{f_\ell^{ne}\} = f_n \{N_n\} + f_s \{N_s\} \quad (14.48-18)$$

It has been shown (Wriggers et al.(137) and Parisch(132)) that a tangent stiffness matrix for contact is formed by the outer product of the interpolation vectors. In general form,

$$[K_\ell] = \begin{cases} K_n \{N_n\} \{N_n\}^T + K_s \{N_s\} \{N_s\}^T & \text{if sticking contact (STAT=1)} \\ K_n \{N_n\} \{N_n\}^T & \text{if sliding or frictionless contact (STAT=2)} \\ [0] & \text{if open contact (STAT=3 or 4)} \end{cases} \quad (14.48-19)$$

Certain terms are modified and added to equation (14.48–19) that are not given in full detail here. These additional terms are those related to adaptive decent as well as certain consistent tangent components.

14.48.5 Thermal/Structural Contact

Combined structural and thermal contact is specified if KEYOPT(1) = 1, which indicates that UX, UY, and TEMP degrees of freedom (DOFs) are active. When contact is established, heat is transferred across the interface in a direction normal to the target surface. The total heat flow from the target surface to the contact node is given as

$$q_n = \begin{cases} K_c (T^* - T_K) & \text{if in contact (STAT} \leq 2) \\ 0 & \text{if open (STAT} > 2) \end{cases} \quad (14.48-20)$$

where:

- K_c = contact conductance (input as COND on **R** command)
- T^* = temperature of the target plane at the contact point
 - = $\frac{1}{2} (1 - s^*) T_I + \frac{1}{2} (1 + s^*) T_J$
- T_I, T_J = current nodal temperatures
- T_K = temperature of contact node K

The thermal conductivity matrix is:

$$[K_\ell^c] = \begin{cases} K_c \{N'_n\} \{N'_n\}^T & \text{if in contact (STAT} \leq 2) \\ [0] & \text{if open (STAT} > 2) \end{cases} \quad (14.48-21)$$

The element thermal load vector is comprised of the Newton–Raphson restoring heat flows, and can be expressed as:

$$\{F_{\ell}^{nr}\} = q_n \{N'_n\} \quad (14.48-22)$$

where:
$$\{N'_n\} = \left[-\frac{1}{2} (1 - s^*) \quad -\frac{1}{2} (1 + s^*) \quad 1 \right]^T$$

The thermal load vector and conductivity matrix are assembled with the structural load vector and stiffness matrix, respectively, in a manner consistent with the defined DOFs.

14.48.6 Description of Element Output Quantities

Several of the variables discussed above appear as output quantities for the CONTAC48 element. These and all other output quantities are summarized below.

- STAT = current status of element. If STAT = 1, sticking contact; if 2, sliding contact; if 3, open but inside pinball (i.e., close); and if 4, open and outside pinball (i.e., far away)
- OLDST = status at the previous time step
- NX, NY = global components of the target surface normal = n (equation (14.48-1))
- FNTOT = total normal force = f_n (equation (14.48-5) or (14.48-6))
- FNPF = Penalty function part of normal force = $K_n g$ ($g < 0$)
- GAP = gap size = g (equation (14.48-3))
- LEN = target length = L
- LOC = normalized location of contact node on target plane = s^* (equation (14.48-4))
- FS = tangential force = f_s (equations (14.48-8), (14.48-11), and (14.48-13))
- FSLIM = Coulomb limit force = \bar{f}_s (equation (14.48-12))
- MU = active friction coefficient = $F\mu$
- ANGLE = principal angle of friction force. ANGLE = 0 if $f_s \geq 0$; ANGLE = 180° if $f_s < 0$
- Q = heat flow at contact (equation (14.48-20))