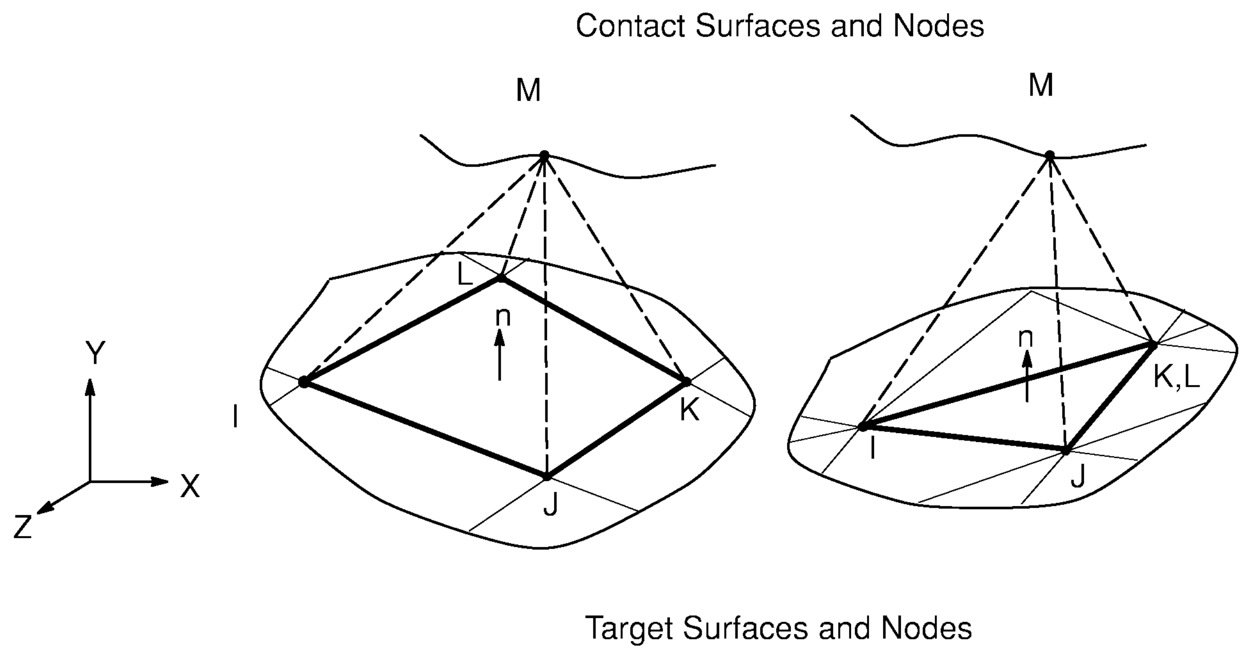
14.49 CONTAC49 — 3–D Point–to–Surface Contact



14.49.1 Introduction

CONTAC49 is a 5–node element that is intended for general contact analysis. In a general contact analysis, the area of contact between two (or more) bodies is generally not known in advance. In addition the finite element models of the contacting bodies are generated is such a way that precise node–to–node contact is neither achievable nor desirable when contact is established. This type of contact situation precludes the use of node–to–node contact elements such as CONTAC52. The CONTAC49 element has the capability to represent general contact of models that are generated with arbitrary meshes. In other words, its use is not limited to known contact or node–to–node configurations.

CONTAC49 is applicable to 3–D geometries. It may be applied to contact of solid bodies or shells, to static or dynamic analyses, to problems with or without friction, and to flexible–to–flexible or rigid–to–flexible body contact. The combined mechanisms of structural contact and thermal contact conductance can also be modeled by CONTAC49.

14.49.2 Contact Kinematics

Contact kinematics is concerned with the precise tracking of contact nodes and surfaces in order to define clear and unambiguous contact conditions. The primary aim is to delineate between open (i.e., not in contact) and closed (in contact) contact situation. This task is accomplished by various algorithms embedded in the CONTAC49 element.

Contact and target definition – With reference to the introductory figure, two potential contact surfaces are referred to as either the “target surface” or the “contact surface”. The “target surface” is represented by “target nodes” I, J, K and L, and the “contact surface” is represented by the “contact node” M. It is usually the case that many CONTAC49 elements will be needed to fully represent a realistic contact problem. (To that end the **GCGEN** command of the PREP7 routine can be used to generate CONTAC49 elements.)

Pinball algorithm – In simple terms, contact occurs whenever the contact node (M) penetrates the target surface (I, J, K, L). The first step in the determination of contact penetration is to make a distinction between near–field and far–field contact. Referring to the 2–D case for simplicity (CONTAC48), Figure 14.48–1 shows several positions of a contact node with respect to the target surface. For CONTAC49 in 3–D the delimiting circle becomes a sphere which is referred to as the “pinball”. When a contact node is outside the pinball an “open” contact condition is assumed, irrespective of whether or not the contact node is above or below the target. Penetration can only occur once the contact node is inside the pinball. The radius of the pinball defaults to be 50% greater than the maximum of the two target surface diagonals and can be overridden by real constant PINB.

Pseudo element algorithm – The next step in the determination of contact is to associate a single target to each contact node depending upon the position of the contact node in space. This is accomplished by establishing solid “pseudo elements” for each target surface as shown in Figure 14.49–1. A unique association is formed whenever contact node M is found within a target’s pseudo element. If a clear distinction is not made it is possible that contact “voids” or “overlaps” can appear (see Figure 14.48–3 in the 2–D case). These voids and overlaps are unavoidable and are due, in the main, to piecewise discretization of surfaces that are actually curved. These solid elements are temporarily formed each equilibrium iteration and provide a continuous mapping for each contact node that is in or nearly in contact with a target. The kinematic information that is needed to build these pseudo elements is stored in a global contact data base that is updated each equilibrium iteration.

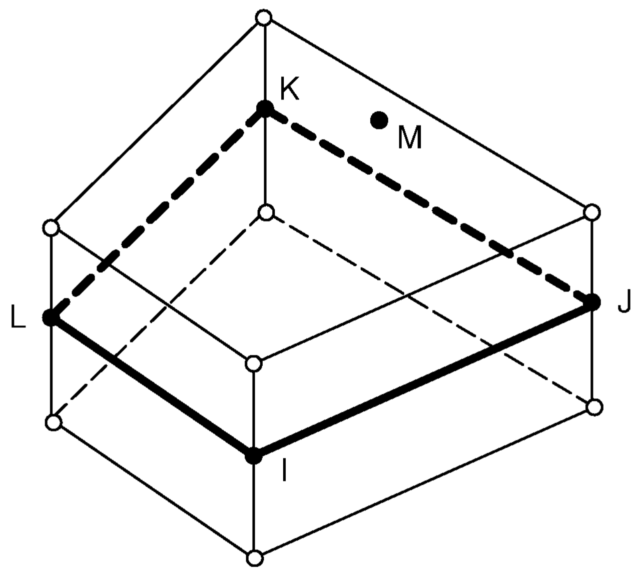


Figure 14.49–1 Pseudo Element

Contact gap and projection – The pinball and pseudo–element algorithms provide a one–to–one mapping between a contact node and a target. The final kinematic step is to determine the open gap or the gap penetration of the contact node on the target plane, along with the point of projection of the contact node. This is achieved by first modifying the target surface nodes to lie in a plane if they do not already, simplifying tangential surface calculations. In other words the warping of the target surface is ignored. In Figure 14.49–2 several coordinate systems are indicated. The global system is the usual X–Y–Z system. The next system is the natural s–t–n system of the planar target surface. A rectangular, invariant, x–y–z system is constructed from the natural s–t–n system in such a way that n– and z–directions are parallel. This enables straightforward tracking of the tangential contact motions. Finally a second rectangular x_e, y_e, z_e system is defined for element force output. Having defined the modified (unwarped) target surface and the various coordinate systems, the contact kinematics of gap and location are left to be defined.

With reference to Figure 14.49–2, the contact location (s*, t*) is computed by an iterative Newton's method based upon a normal projection of the contact node to the target plane. At the projected contact point a value of gap (g) is determined by the contact node's location with respect to the target plane. Contact penetration is assumed to occur if the value of g is found to be negative, and the s* and t* projections are found to be in the natural space bounds of the target. For the later condition, the target surface is internally expanded if input quantity TOLS on **R** command is specified, thereby increasing the chances that a contact node will come into contact with the target plane. A positive gap value indicates an open contact condition.

These various conditions of contact are referred to in CONTAC49's output as "status". They are:

STAT (or OLDST) = 4	open and outside the pinball
STAT (or OLDST) = 3	open and inside the pinball
STAT (or OLDST) < 3	contact has occurred (STAT=1 or 2 are described below)

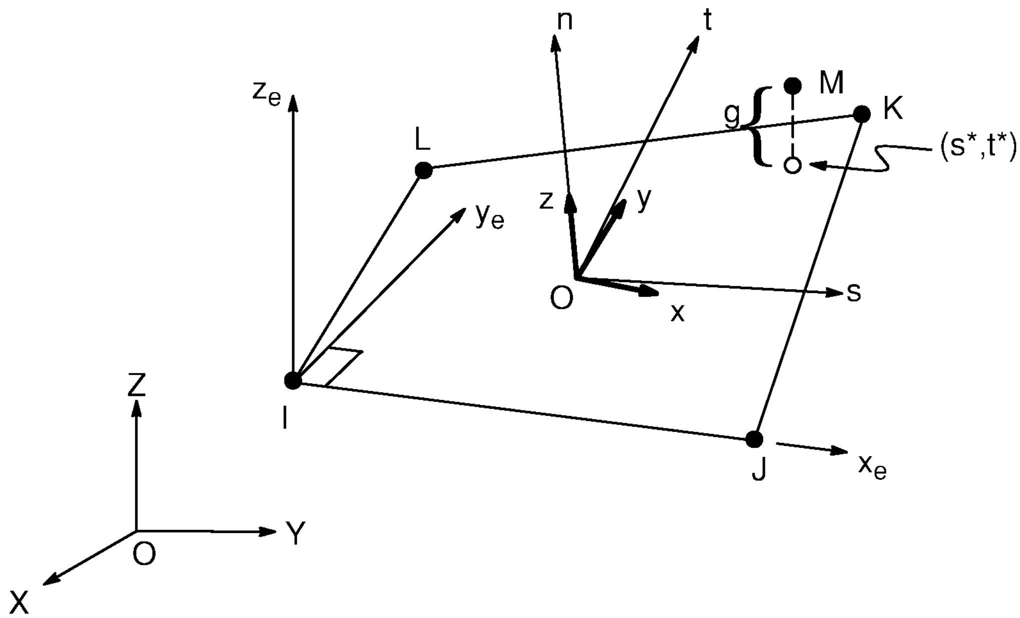


Figure 14.49–2 Target Coordinate Systems

14.49.3 Contact Forces

As explained above, contact is indicated when the contact node M penetrates the target surface defined by target nodes I, J, K, and L. This penetration is represented by the magnitude of the gap (g) and is a violation of compatibility. In order to satisfy contact compatibility, forces are developed in a direction normal (n–direction) to the target that will tend to reduce the penetration to an acceptable numerical level. In addition to compatibility forces, friction forces are developed in directions that are tangent to the target plane. The normal and tangential friction forces that are described here are referenced to the local x–y–z system shown in Figure 14.49–2.

Normal forces – Two methods of satisfying contact compatibility are available for CONTAC49: a penalty method (KEYOPT(2)=0) or a combined penalty plus Lagrange multiplier method (KEYOPT(2)=1). The penalty method approximately enforces compatibility by means of a contact stiffness (i.e., the penalty parameter). The combined approach satisfies compatibility to a user–defined precision by the generation of additional contact forces (i.e., Lagrange forces).

For the penalty method,

$$f_n = \begin{cases} K_n g & \text{if } g \le 0 \\ 0 & \text{if } g > 0 \end{cases} \qquad (14.49\text{–}1)$$

where: K_n = contact stiffness (input quantity KN on **R** command)

For the combined method, the Lagrange multipler component of force is computed locally (for each element) and iteratively. It is expressed as

$$f_n = \min\,(0,\ K_n g + \lambda_{i+1}) \qquad (14.49\text{–}2)$$

where: λ_{i+1} = Lagrange multiplier force at iteration i+1

$$= \begin{cases} \lambda_i + \alpha K_n g & \text{if } |\,g\,| \ \ge \varepsilon \\ \lambda_i & \text{if } |\,g\,| \ < \varepsilon \end{cases} \qquad (14.49\text{–}3)$$

ε = user–defined compatibility tolerance (input quantity TOLN on **R** command)

α = an internally computed factor ($\alpha < 1$)

Tangential forces – Tangential forces are due to friction that arises as the contact node meets and moves along the target. The CONTAC49 element considers three friction models: frictionless (KEYOPT(3)=0), elastic Coulomb friction (KEYOPT(3)=1), and rigid Coulomb friction (KEYOPT(3)=2). The Coulomb friction representations requires the specification of the coefficient of sliding friction (μ), which is supplied by means of material property MU on the **MP** command.

For the frictionless case (KEYOPT(3)=0) the tangential force is merely:

$$f_x = f_y = 0 \qquad (14.49\text{–}4)$$

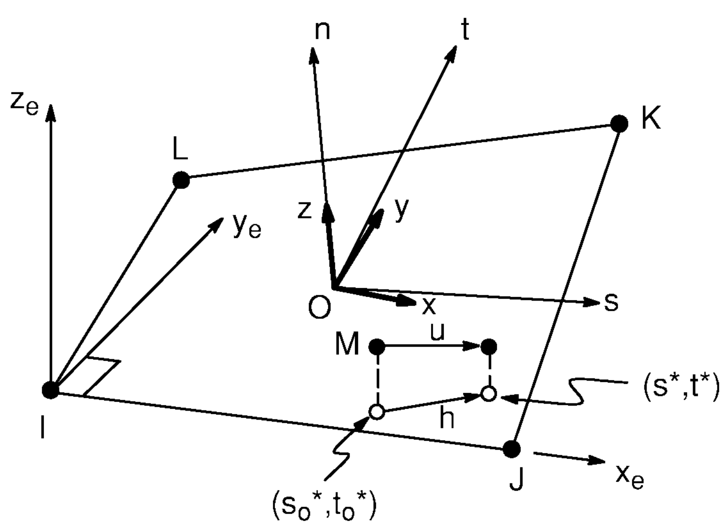


Figure 14.49–3 Location of Contact Node on the Target Plane

For elastic Coulomb friction (KEYOPT(3)=1) it is necessary to calculate the tangential deformations of the contact node relative to the target. Figure 14.49–3 shows the total motion (u) of contact node M along the target plane. It is seen that the total tangential displacement (η) is represented by the projection of the total contact node motion to the unwarped plane of the target. Two projection points are mapped in natural coordinates (s, t). The point (s^*, t^*) is the current projection position, and the tangential deformation is tracked from the point (s_o^*, t_o^*) that is associated with the previous converged solution (i.e., the previous time point). The deformation is first separated into x and y components, such that

$$\eta = \left(\eta_x^2 + \eta_y^2\right)^{1/2} \tag{14.49–5}$$

where:

η_x = component of η in the local x direction

η_y = component of η in the local y direction

Next, the deformation is decomposed into elastic (or sticking) and sliding (or inelastic) components.

$$\begin{aligned} \eta_x &= \eta_x^c + \eta_x^s \\ \eta_y &= \eta_y^c + \eta_y^s \end{aligned} \tag{14.49–6}$$

Related tangential forces are:

$$\begin{aligned} f_x &= K_t\, \eta_x^c \\ f_y &= K_t\, \eta_y^c \end{aligned} \tag{14.49–7}$$

where:

K_t = sticking stiffness (input quantity KT on **R** command)

It follows that the magnitude of the tangential forces is

$$f_s = \left(f_x^2 + f_y^2\right)^{1/2} \tag{14.49–8}$$

The delineation between sticking and sliding conditions is expressed by:

$$f_s = \bar{f}_s, \quad \text{if sliding (STAT = 2)} \tag{14.49–9}$$

$$f_s < F\bar{f}_s, \quad \text{if sticking (STAT = 1)} \tag{14.49–10}$$

where:

F = static/dynamic friction factor (input quantity FACT on **R** command)

The limiting sliding force in the Coulomb model is

$$\bar{f}_s = -\mu \, f_n \tag{14.49–11}$$

Equations (14.49–1) thru (14.49–11) are merely a summary of contact forces and displacements. The actual computation that is performed uses a technique that is similar to that of non–associative theory of plasticity (see Section 4.1). In each substep that sliding friction occurs, an elastic predictor is computed in contact traction space (Figure 14.49–4). The predictor is modified via a radial return mapping function, providing both a small elastic deformation along with sliding response as developed by Giannakopoulos(135).

For the elastic Coulomb model, initial contact is always treated as elastic sticking (STAT=1), but with the tangential force set to zero (f_s=0). In other words, the goal of initial contact is intentionally limited to the determination of the penetration (g) and the contact point (s^*, t^*), irrespective of friction forces. All subsequent substeps will allow friction to develop according to Equations (14.49–9) and (14.49–10).

Turning attention to the rigid Coulomb model of friction (KEYOPT(3)=2), elastic contact deformations is ignored. If the contact node M is penetrating the target, it is always assumed to be sliding (STAT=2). Tangential forces are

$$f_x = \frac{\eta_x}{\eta} \bar{f}_s \qquad f_y = \frac{\eta_y}{\eta} \bar{f}_s \tag{14.49–12}$$

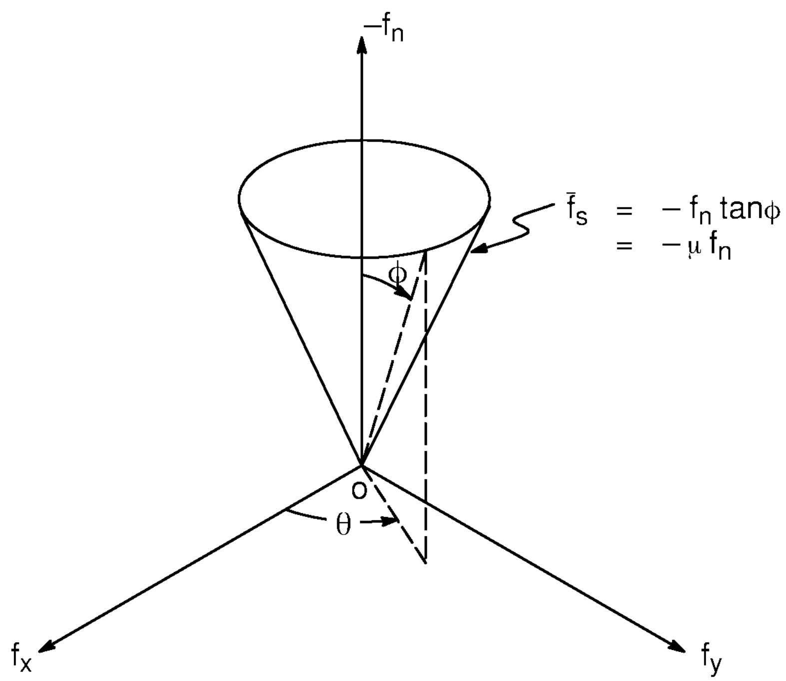


Figure 14.49–4 Contact Friction Space for Coulomb Friction

Contact force transition – A special situation arises when a contact node moves from one target to another. When this occurs, the contact history is passed from the target that was in contact to the target that is currently subjected to contact. In so doing, the path–dependence of friction is maintained and, for some problems, convergence behavior is seen to improve. The transition makes use of a contact database that contain contact conditions and forces for all contact nodes in actual contact.

14.49.4 Stiffness Matrix And Load Vector

It is convenient to define three interpolation vectors in terms of the local s–t coordinates. These interpolation vectors are evaluated at the point of projection ($s=s^*$, and $t=t^*$) of the contact node M to the target plane (see Figure 14.49–2). (Note that s and t are dimensionless coordinates, ranging from –1 to 1 for quadrilateral targets and 0 to 1 for triangular targets.)

$$\{N_n\}^T = \lfloor 0 \quad 0 \quad q_1 \quad 0 \quad 0 \quad q_2 \quad 0 \quad 0 \quad q_3 \quad 0 \quad 0 \quad q_4 \quad 0 \quad 0 \quad 1 \rfloor \tag{14.49–13}$$

$$\{N_x\}^T = \lfloor q_1 \quad 0 \quad 0 \quad q_2 \quad 0 \quad 0 \quad q_3 \quad 0 \quad 0 \quad q_4 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \rfloor \tag{14.49–14}$$

$$\{N_y\}^T = \lfloor 0 \quad q_1 \quad 0 \quad 0 \quad q_2 \quad 0 \quad 0 \quad q_3 \quad 0 \quad 0 \quad q_4 \quad 0 \quad 0 \quad 1 \quad 0 \rfloor \tag{14.49–15}$$

For the 4–node target, individual interpolates are

$$\begin{aligned} q_1 &= -\frac{1}{4}(1 - s^*)(1 - t^*) \\ q_2 &= -\frac{1}{4}(1 + s^*)(1 - t^*) \\ q_3 &= -\frac{1}{4}(1 + s^*)(1 + t^*) \\ q_4 &= -\frac{1}{4}(1 - s^*)(1 + t^*) \end{aligned} \tag{14.49–16}$$

And for the 3–node target

$$\begin{aligned} q_1 &= -s^* \\ q_2 &= -t^* \\ q_3 &= -1 + s^* + t^* \\ q_4 &= 0 \end{aligned} \tag{14.49–17}$$

Figure 14.48–7 shows all nodal contact forces for the CONTAC48 element, the 3–node 2–D contact element. A similar representation exists for both the 5–node and 4–node CONTAC49 element. In the normal direction, the force applied to the contact node (M) is defined in equations (14.49–1) and (14.49–3) and is balanced by opposite forces applied to the target nodes; that is,

$$f_{n,M} = f_{n,I} + f_{n,J} + f_{n,K} + f_{n,L} = f_n \tag{14.49–18}$$

Similarly, in the tangential directions,

$$\begin{aligned} f_{x,M} &= f_{x,I} + f_{x,J} + f_{x,K} + f_{x,L} = f_x \\ f_{y,M} &= f_{y,I} + f_{y,J} + f_{y,K} + f_{y,L} = f_y \end{aligned} \tag{14.49–19}$$

where f_x and f_y are defined in the previous subsection.

Using the interpolation vector above, the element load vector (i.e., the Newton–Raphson restoring forces) is:

$$\left\{F_\ell^{nc}\right\} = f_n \{N_n\} + f_x \{N_x\} + f_y \{N_y\} \tag{14.49–20}$$

It has been shown (Wriggers et al.(137), Stein et al.(138)) that a tangent stiffness matrix for contact is formed by the outer product of the interpolation vectors. In general form,

$$[K_\ell] = \begin{cases} K_n \{N_n\}\{N_n\}^T + K_s(\{N_x\}\{N_x\}^T + \{N_y\}\{N_y\}^T), & \text{if sticking contact (STAT=1)} \\ K_n \{N_n\}\{N_n\}^T & \text{if sliding or frictionless contact (STAT=2)} \\ [0] & \text{if open contact (STAT=3 or 4)} \end{cases} \tag{14.49–21}$$

Certain terms are modified and added to Equation (14.49–21) that are not given in full detail here. These additional terms are those related to adaptive decent as well as certain internal element manipulations.

14.49.5 Thermal/Structural Contact

Combined structural and thermal contact is specified if KEYOPT(1) = 1, which indicates that UX, UY, and TEMP degrees of freedom (DOFs) are active. When contact is established, heat is transferred across the interface in a direction normal to the target surface. The total heat flow from the target surface to the contact node is given as

$$q_n = \begin{cases} K_c\,(T^* - T_M) & \text{if in contact (STAT} \le 2) \\ 0 & \text{if open (STAT} > 2) \end{cases} \quad (14.49\text{–}22)$$

where:
- K_c = contact conductance (input quantity COND on **R** command)
- T^* = temperature of the target plane at the contact point
  = $-\,q_1T_I - q_2T_J - q_3T_K - q_4T_L$
- T_I, T_J, T_K, T_L = current temperatures of the target nodes
- T_M = temperature of contact node M

The thermal conductivity matrix is:

$$\left[K_\ell^c\right] = \begin{cases} K_c\,\{N_n'\}\,\{N_n'\}^T & \text{if in contact (STAT} \le 2) \\ [0] & \text{if open (STAT} > 2) \end{cases} \quad (14.49\text{–}23)$$

The element thermal load vector is comprised of the Newton–Raphson restoring heat flows, and can be expressed as:

$$\{F_\ell^{nr}\} = q_n\,\{N_n'\} \quad (14.49\text{–}24)$$

where:
$$\{N_n'\} = \lfloor q_1 \quad q_2 \quad q_3 \quad q_4 \quad 1 \rfloor^T$$

The thermal load vector and conductivity matrix are assembled with the structural load vector and stiffness matrix, respectively, in a manner consistent with the defined DOFs.

14.49.6 Description Of Element Output Quantities

Several of the variables discussed above appear as output quantities for the CONTAC49 element. These and all other output quantities are summarized below.

- STAT = current status of element. If STAT = 1, sticking contact; if 2, sliding contact; if 3, open but inside pinball (i.e., close); and if 4, open and outside pinball (i.e., far away)
- OLDST = status at the previous time step
- NX,NY,NZ = global x–y–z components of the target surface normal
- FNTOT = total normal force = f_n (Equation (14.49–1) or (14.49–2))
- FNPF = Penalty function part of normal force = $K_n g$ ($g < 0$) (Equation (14.49–1))
- GAP = gap size = g
- AREA = target area

LOC1,LOC2	=	normalized locations of contact node on target plane = s^* and t^*
FS1,FS2	=	tangential forces that are rotated into element (x_e, y_e, z_e). FS1 is in the x_e direction, and FS2 in y_e (see Figure 14.49–2)
FSLIM	=	Coulomb limit force = $\bar{f}_s$ (Equation (14.49–11))
MU	=	active friction coefficient = $F\mu$
ANGLE	=	$\theta = \tan^{-1} (f_y / f_x)$ principal angle of friction force (see Figure 14.49–4)
Q	=	heat flow at contact (equation (14.49–22))