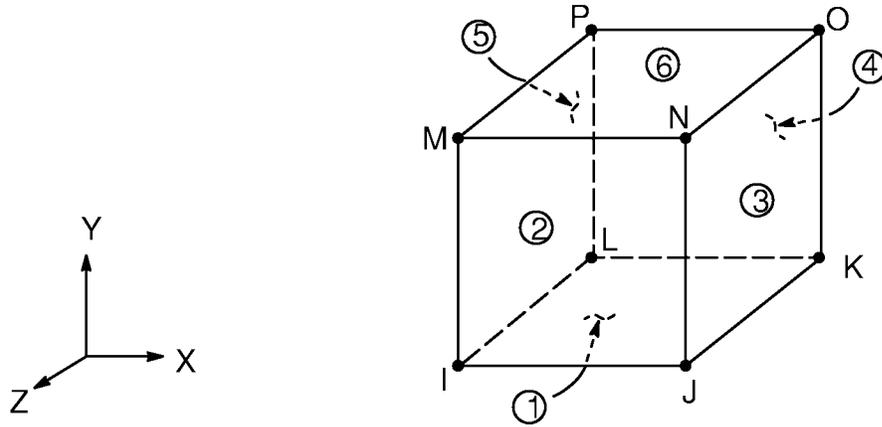


# 14.80 FLUID80 — 3-D Contained Fluid



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equation (12.8.18-1), (12.8.18-2), and (12.8.18-3)	1 x 1 x 1 for bulk strain effects. 2 x 2 x 2 for shear and rotational resistance effects
Mass Matrix	Same as stiffness matrix. Matrix is diagonalized as described in Section 13.2	2 x 2 x 2
Damping Matrix	Same as stiffness matrix	2 x 2 x 2
Temperature Load Vector	Same as stiffness matrix	1 x 1 x 1
Pressure Load Vector	Same as stiffness matrix, specialized to the face	2 x 2

Load Type	Distribution
Element Temperature	Average of the 8 nodal temperatures is used throughout element
Nodal Temperature	Average of the 8 nodal temperatures is used throughout element
Pressure	Bilinear across each face

### 14.80.1 Other Applicable Sections

Chapter 2 describes the derivation of element matrices and load vectors. Section 13.1 describes integration point locations.

### 14.80.2 Assumptions and Restrictions

This element does not generate a consistent mass matrix; only the lumped mass matrix is available.

### 14.80.3 Material Properties

Rather than equation (2.1–3), the stress–strain relationships used to develop the stiffness matrix and thermal load vector are:

$$\begin{pmatrix} \epsilon_{\text{bulk}} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \\ R_x \\ R_y \\ R_z \end{pmatrix} = \begin{pmatrix} 3\alpha\Delta T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{bmatrix} \frac{1}{K} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{S} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{S} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{S} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{B} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{B} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{B} \end{bmatrix} \begin{pmatrix} P \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \\ M_x \\ M_y \\ M_z \end{pmatrix} \quad (14.80-1)$$

where:

$$\epsilon_{\text{bulk}} = \text{bulk strain} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$\alpha$  = thermal coefficient of expansion (input as ALPX on **MP** command)

- $\Delta T$  = change of temperature from reference temperature  
 $K$  = fluid elastic (bulk) modulus (input quantity EX on **MP** command)  
 $P$  = pressure  
 $\gamma$  = shear strain  
 $S$  =  $K \times 10^{-9}$  (arbitrarily small number to give element some shear stability)  
 $\tau$  = shear stress  
 $R_i$  = rotation about axis  $i$   
 $B$  =  $K \times 10^{-9}$  (arbitrarily small number to give element some rotational stability)  
 $M_i$  = twisting force about axis  $i$

A damping matrix is also developed based on:

$$\begin{pmatrix} \dot{\epsilon}_{\text{bulk}} \\ \dot{\gamma}_{xy} \\ \dot{\gamma}_{yz} \\ \dot{\gamma}_{zx} \\ \dot{R}_x \\ \dot{R}_y \\ \dot{R}_z \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\eta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\eta} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\eta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{c} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{c} \end{bmatrix} \begin{pmatrix} P \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \\ M_x \\ M_y \\ M_z \end{pmatrix} \quad (14.80-2)$$

where:

$\eta$  = viscosity (input as VISC on **MP** command)  
 $c$  =  $.00001 * \eta$

and the  $(\cdot)$  represents differentiation with respect to time.

A lumped mass matrix is developed, based on the density (input as DENS on **MP** command).

## 14.80.4 Free Surface Effects

The free surface is handled with an additional special spring effect. The necessity of these springs can be seen by studying a U-Tube, as shown in Figure 14.80-1.

Note that if the left side is pushed down a distance of  $\Delta h$ , the displaced fluid mass is:

$$M_D = \Delta h A \rho \quad (14.80-3)$$

where:

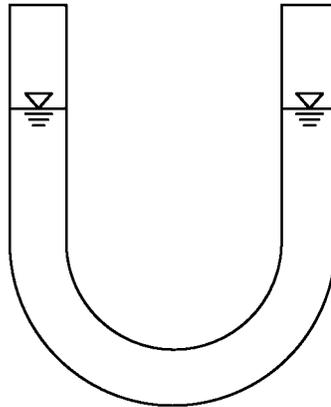
- $M_D$  = mass of displaced fluid
- $\Delta h$  = distance fluid surface has moved
- $A$  = cross-sectional area of U-Tube
- $\rho$  = fluid density

Then, the force required to hold the fluid in place is

$$F_D = M_D g \quad (14.80-4)$$

where:

- $F_D$  = force required to hold the fluid in place
- $g$  = acceleration due to gravity (input on **ACEL** command)



**Figure 14.80-1 U-Tube with Fluid**

Finally, the stiffness at the surface is the force divided by the distance, or

$$K_s = \frac{F_D}{\Delta h} = \rho A g \quad (14.80-5)$$

This expression is generalized to be:

$$K_s = \rho A_F (g_x C_x + g_y C_y + g_z C_z) \quad (14.80-6)$$

where:

- $A_F$  = area of the face of the element
- $g_i$  = acceleration in the  $i$  direction
- $C_i$  =  $i$ th component of the normal to the face of the element

This results in adding springs from each node to ground, with the spring constants being positive on the top of the element, and negative on the bottom. For an interior node, positive and negative effects cancel out and, at the bottom where the boundary

must be fixed to keep the fluid from leaking out, the negative spring has no effect. If KEYOPT(2)=1, positive springs are added only to faces located at  $z = 0.0$ .

### 14.80.5 Other Assumptions and Limitations

The surface springs tend to retard the hydrostatic motions of the element from their correct values. The hydrodynamic motions are not changed. From the definition of bulk modulus,

$$u_s = \int_0^H \frac{P}{K} dz \quad (14.80-7)$$

where:

- $u_s$  = vertical motion of a static column of fluid (unit cross-sectional area)
- $H$  = height of fluid column
- $P$  = pressure at any point
- $z$  = distance from free surface

The pressure is normally defined as:

$$P = \rho g z \quad (14.80-8)$$

But this pressure effect is reduced by the presence of the surface springs, so that

$$P = \rho g z - K_s u_s = \rho g (z - u_s) \quad (14.80-9)$$

Combining equations (14.80-7) and (14.80-9) and integrating,

$$u_s = \frac{\rho g}{K} \left( \frac{H^2}{2} - u_s H \right) \quad (14.80-10)$$

or

$$u_s = \frac{1}{1 - \frac{H \rho g}{K}} \frac{\rho g}{K} \frac{H^2}{2} \quad (14.80-11)$$

If there were no surface springs,

$$u_s = \frac{\rho g}{K} \frac{H^2}{2} \quad (14.80-12)$$

Thus the error for hydrostatic effects is the departure from 1.0 of the factor  $(1 / (1 - H_0 g / K))$ , which is normally quite small.

The 1 x 1 x 1 integration rule is used to permit the element to “bend” without the bulk modulus resistance being mobilized, i.e.



**Figure 14.80–2 “Bending Without Resistance”**

While this motion is permitted, other motions in a static problem often result, which can be thought of as energy-free eddy currents. For this reason, small shear and rotational resistances are built in, as indicated in equation (14.80–1).