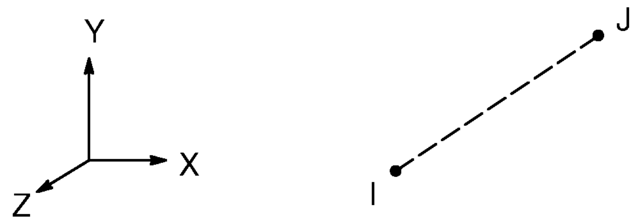
14.31 LINK31 — Radiation Link



Matrix or Vector	Shape Functions	Integration Points
Conductivity Matrix	None used (nodes may be coincident)	None

14.31.1 Standard Radiation (KEYOPT(3) = 0)

The two–surface radiation equation (from equation (6.1–12)) that is solved (iteratively) is:

$$Q = \sigma \epsilon FA \left(T_I^4 - T_J^4\right) \tag{14.31–1}$$

where:
- Q = heat flow rate between nodes I and J (output quantity HEAT RATE)
- σ = Stefan–Boltzmann constant (input as SBC on **R** command)
- ε = emissivity (input as EMISSIVITY on **R** or EMIS on **MP** command)
- F = geometric form factor (input as FORM FACTOR on **R** command)
- A = area of element (input as AREA on **R** command)
- T_I, T_J = absolute temperatures at nodes I and J

The program uses a linear equation solver. Therefore, equation (14.31–1) is expanded as:

$$Q = \sigma \epsilon FA \left(T_I^2 + T_J^2\right)(T_I + T_J)(T_I - T_J) \tag{14.31–2}$$

and then rewritten as:

$$Q = \sigma \epsilon FA \left(T_{I,n-1}^2 + T_{J,n-1}^2\right)\left(T_{I,n-1} + T_{J,n-1}\right)\left(T_{I,n} - T_{J,n}\right) \tag{14.31–3}$$

where the subscripts n and n–1 refer to the current and previous iterations, respectively. It is then recast into finite element form:

$$\begin{Bmatrix} Q_I \\ Q_J \end{Bmatrix} = C_o \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_{I,n} \\ T_{J,n} \end{Bmatrix} \tag{14.31–4}$$

with

$$C_o = \sigma \epsilon FA \left(T^2_{I,n-1} + T^2_{J,n-1}\right)\left(T_{I,n-1} + T_{J,n-1}\right) \tag{14.31–5}$$

14.31.2 Empirical Radiation (KEYOPT(3) = 1)

The basic equation is:

$$Q = \sigma \epsilon \left(FT^4_I - AT^4_J\right) \tag{14.31–6}$$

instead of equation (14.31–1). This form leads to

$$C_o = \sigma \epsilon \left(F^{\frac{1}{2}} T^2_{I,n-1} + A^{\frac{1}{2}} T^2_{J,n-1}\right)\left(F^{\frac{1}{4}} T_{I,n-1} + A^{\frac{1}{4}} T_{J,n-1}\right) \tag{14.31–7}$$

instead of equation (14.31–5). And, hence the matrix equation (14.31–4) becomes:

$$\begin{Bmatrix} Q_I \\ Q_J \end{Bmatrix} = C_o \begin{bmatrix} F^{\frac{1}{4}} & -A^{\frac{1}{4}} \\ -F^{\frac{1}{4}} & A^{\frac{1}{4}} \end{bmatrix} \begin{Bmatrix} T_{I,n} \\ T_{J,n} \end{Bmatrix} \tag{14.31–8}$$

14.31.3 Solution

If the emissivity is input on a temperature dependent basis, equation (14.31–5) is rewritten to be:

$$C_o = \sigma FA \left(\beta^2_{I,n-1} + \beta^2_{J,n-1}\right)\left(\beta_{I,n-1} + \beta_{J,n-1}\right) \tag{14.31–9}$$

where:

$\beta_i = T_i \left(\epsilon_i\right)^{\frac{1}{3}}$ (i = I or J)

ε_i = emissivity at node i evaluated at temperature T^f_i

$$T_i^f = T_i - T_{off}$$

T_{off} = offset temperature (input on **TOFFST** command)

Equation (14.31–7) is handled analogously.