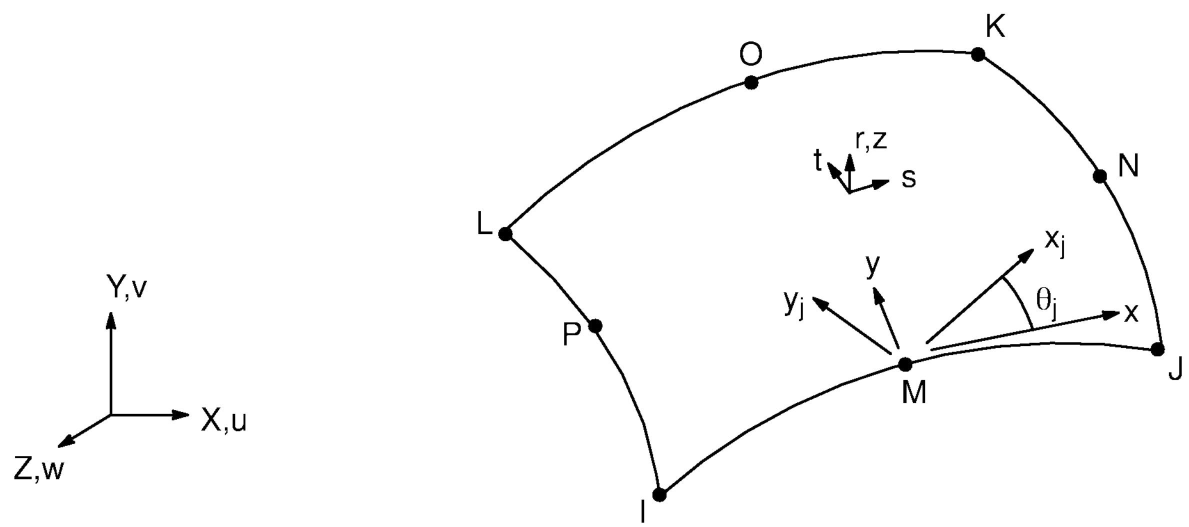
14.91 SHELL91 — Nonlinear Layered Structural Shell



Matrix or Vector	Geometry	Shape Functions	Integration Points
Stiffness Matrix	Quad	Equations (12.5.14–1), (12.5.14–2), and (12.5.14–3)	Thru–the–thickness: 3 for each layer In–plane: 2 x 2
	Triangle	Equations (12.5.5–1), (12.5.5–2), and (12.5.5–3)	Thru–the–thickness: 3 for each layer In–plane: 3
Mass Matrix	Quad	Equations (12.5.10–1), (12.5.10–2), and (12.5.10–3)	Same as stiffness matrix
	Triangle	Equations (12.5.2–1), (12.5.2–2), and (12.5.2–3)	Same as stiffness matrix
Stress Stiffness Matrix	Same as mass matrix		Same as stiffness matrix

Matrix or Vector	Geometry	Shape Functions	Integration Points
Thermal Load Vector	Same as stiffness matrix		Same as stiffness matrix
Transverse Pressure Load Vector	Quad	Equation (12.5.10–3)	2 x 2
	Triangle	Equation (12.5.2–3)	3
Edge Pressure Load Vector	Same as in–plane mass matrix specialized to the edge.		2

Load Type	Distribution
Element Temperature	Linear thru each layer, bilinear in plane of element
Nodal Temperature	Constant thru thickness, bilinear in plane of element
Pressure	Bilinear in plane of element, linear along each edge

Reference: Ahmad(1), Cook(5)

14.91.1 Other Applicable Sections

Chapter 2 describes the derivation of structural element matrices and load vectors as well as stress evaluations. Section 13.1 describes integration point locations. The mass matrix is diagonalized as described in Section 13.2. Section 14.99 describes the failure criterion.

14.91.2 Assumptions and Restrictions

Normals to the centerplane are assumed to remain straight after deformation, but not necessarily normal to the centerplane.

Each pair of integration points (in the r direction) is assumed to have the same element (material) orientation.

There is no significant stiffness associated with rotation about the element r axis. A nominal value of stiffness is present using the approach of Zienkiewicz(39), however, to prevent free rotation at the node.

This element does not generate a consistent mass matrix; only the lumped mass matrix is available.

14.91.3 Stress–Strain Relationship

The material property matrix $[D]_j$ for the layer j is:

$$[D]_j = \begin{bmatrix} BE_{x,j} & B\nu_{xy,j}E_{x,j} & 0 & 0 & 0 & 0 \\ B\nu_{xy,j}E_{x,j} & BE_{y,j} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{xy,j} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{G_{yz,j}}{f} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{G_{xz,j}}{f} \end{bmatrix} \quad (14.91\text{–}1)$$

where:

$B = \dfrac{E_{y,j}}{E_{y,j} - (\nu_{xy,j})^2 E_{x,j}}$

$E_{x,j}$ = Young's modulus in layer x direction of layer j (input as EX on **MP** command)

$\nu_{xy,j}$ = Poisson's ratio in layer x–y plane of layer j (input as NUXY on **MP** command)

$G_{xy,j}$ = shear modulus in layer x–y plane of layer j (input as GXY on **MP** command)

$f = \begin{Bmatrix} 1.2 \\ 1.0 + .2 \dfrac{A}{25t^2} \end{Bmatrix}$, whichever is greater

A = element area (in s–t plane)

t = average total thickness

The above definition of f is designed to avoid shear locking. Unlike most other elements, the temperature–dependent material properties are evaluated at each of the in–plane integration points, rather than only at the centroid.

14.91.4 Stress, Force and Moment Calculations

The shape functions assume that the transverse shear strains are constant thru the thickness. However, these strains must be zero at the free surface. Therefore, unless nonlinear materials are used or the sandwich option is used (KEYOPT(9) = 1), they are adjusted by:

$$\sigma'_{xz,j} = \frac{3}{2}\left(1 - r^2\right) \sigma_{xz,j} \tag{14.91–2}$$

$$\sigma'_{yz,j} = \frac{3}{2}\left(1 - r^2\right) \sigma_{yz,j} \tag{14.91–3}$$

where typically:

$\sigma'_{xz,j}$ = adjusted value of transverse shear stress

$\sigma_{xz,j}$ = transverse shear stress as computed from strain–displacement relationships

r = normal coordinate, varying from –1.0 (bottom) to +1.0 (top)

Even with this adjustment, these strains will not be exact due to the variable nature of the material properties thru the thickness. However, for thin shell environments, these strains and their resulting stresses are small in comparison to the x, y, and xy components. The interlaminar shear stresses are equivalent to the transverse shear stresses at the layer boundaries and are computed using equilibrium considerations, and hence are more accurate for most applications.

Force and Moment Summations

The in–plane forces are computed as:

$$T_x = \sum_{j=1}^{N_\ell} t_j \left[\frac{\sigma^t_{x,j} + \sigma^b_{x,j}}{2}\right] \tag{14.91–4}$$

$$T_y = \sum_{j=1}^{N_\ell} t_j \left[\frac{\sigma^t_{y,j} + \sigma^b_{y,j}}{2}\right] \tag{14.91–5}$$

$$T_{xy} = \sum_{j=1}^{N_\ell} t_j \left[\frac{\sigma^t_{xy,j} + \sigma^b_{xy,j}}{2}\right] \tag{14.91–6}$$

where typically:

T_x = output quantity

N_ℓ = number of layers

$\sigma^t_{x,j}$ = stress at top of layer j in element x direction

$\sigma^b_{x,j}$ = stress at bottom of layer j in element x direction

t_j = thickness of layer j

The out–of–plane moments are computed as:

$$M_x = \frac{1}{6}\sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{x,j}\left(2z^b_j + z^t_j\right) + \sigma^t_{x,j}\left(2z^t_j + z^b_j\right)\right) \qquad (14.91\text{–}7)$$

$$M_y = \frac{1}{6}\sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{y,j}\left(2z^b_j + z^t_j\right) + \sigma^t_{y,j}\left(2z^t_j + z^b_j\right)\right) \qquad (14.91\text{–}8)$$

$$M_{xy} = \frac{1}{6}\sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{xy,j}\left(2z^b_j + z^t_j\right) + \sigma^t_{xy,j}\left(2z^t_j + z^b_j\right)\right) \qquad (14.91\text{–}9)$$

where, typically:
- M_x = output quantity MX
- z^b_j = z coordinate of bottom layer of j
- z^t_j = z coordinate of top of layer j
- z = coordinate normal to shell, with z=0 being at shell midsurface

The transverse shear forces are computed as:

$$N_x = \sum_{j=1}^{N_\ell} t_j\, \sigma_{xz,j} \qquad (14.91\text{–}10)$$

$$N_y = \sum_{j=1}^{N_\ell} t_j\, \sigma_{yz,j} \qquad (14.91\text{–}11)$$

where, typically:
- N_x = output quantity NX
- $\sigma_{xz,j}$ = average transverse shear stress in layer j in element x–z plane

For this computation of transverse shear forces, the shear stresses have not been adjusted as shown in equation (14.91–2) and (14.91–3).

Interlaminar Shear Stress Calculation

In the absence of body forces, the in–plane equilibrium equations of infinitesimally small volume are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \qquad (14.91\text{–}12)$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{y}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \qquad (14.91\text{–}13)$$

Rewriting these in incremental form,

$$\Delta\sigma_{xz} = -\Delta z\left(\frac{\Delta\sigma_{x}}{\Delta x} + \frac{\Delta\sigma_{xy}}{\Delta y}\right) \qquad (14.91\text{–}14)$$

$$\Delta\sigma_{yz} = -\Delta z\left(\frac{\Delta\sigma_{yx}}{\Delta x} + \frac{\Delta\sigma_{y}}{\Delta y}\right) \qquad (14.91\text{–}15)$$

Setting these equations in terms of layer j,

$$\Delta\sigma_{xz,j} = -t_j\left(\frac{\Delta\sigma_{x,j}}{\Delta x} + \frac{\Delta\sigma_{xy,j}}{\Delta y}\right) \qquad (14.91\text{–}16)$$

$$\Delta\sigma_{yz,j} = -t_j\left(\frac{\Delta\sigma_{yx,j}}{\Delta x} + \frac{\Delta\sigma_{y,j}}{\Delta y}\right) \qquad (14.91\text{–}17)$$

where:

$\Delta\sigma_{x,j} = \left(\sigma^2_{x,j} + \sigma^3_{x,j} - \sigma^1_{x,j} - \sigma^4_{x,j}\right)/2$

$\Delta\sigma_{xy,j} = \left(\sigma^4_{xy,j} + \sigma^3_{xy,j} - \sigma^1_{xy,j} - \sigma^2_{xy,j}\right)/2$

$\Delta\sigma_{yx,j} = \left(\sigma^2_{xy,j} + \sigma^3_{xy,j} - \sigma^1_{xy,j} - \sigma^4_{xy,j}\right)/2$

$\Delta\sigma_{y,j} = \left(\sigma^4_{y,j} + \sigma^3_{y,j} - \sigma^1_{y,j} - \sigma^2_{y,j}\right)/2$

$\sigma^3_{x,j}$ = stress in element x direction in layer j at integration point 3

Δx and Δy are shown in Figure 14.91–1.

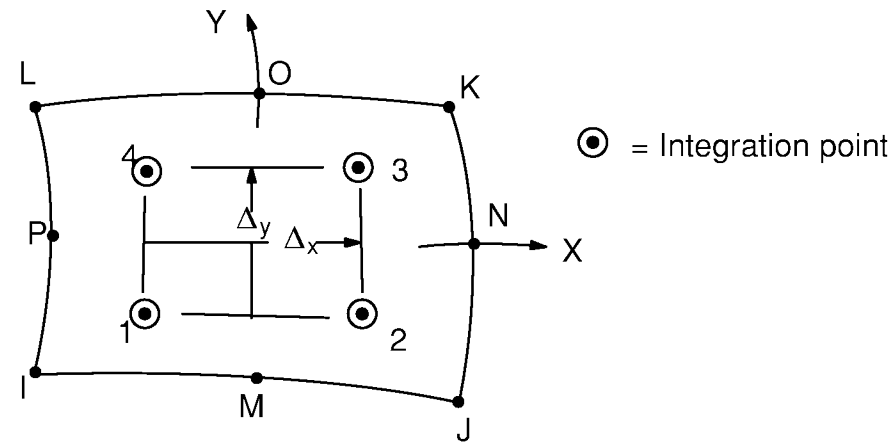


Figure 14.91–1 Integration Point Locations

Thus, the interlaminar shear stress is:

$$\tau_x^k = \sum_{j=1}^{k} \Delta\sigma_{xz,j} - S_x \sum_{j=1}^{k} t_j \tag{14.91–18}$$

$$\tau_y^k = \sum_{j=1}^{k} \Delta\sigma_{yz,j} - S_y \sum_{j=1}^{k} t_j \tag{14.91–19}$$

where, typically, τ_x^k = interlaminar shear stress between layers k and k+1 (output quantity ILSXZ)

$S_x = \dfrac{\sum_{j=1}^{N\ell} \Delta\sigma_{xz,j}}{t}$ (= correction term)

t = total thickness

14.91.5 Sandwich Option

If KEYOPT(9) = 1, SHELL91 uses “sandwich ” logic. This causes:

a. The term f in equation (14.91–1) to be set to 1.0 for the middle layer (core).

b. The transverse shear moduli (G_{yz} and G_{xz}) are set to zero for the top and bottom layers.

c. The transverse shear strains and stresses in the face plate (non–core) layers are set to 0.0.

d. As mentioned earlier, the adjustment to the transverse shear strains and stresses in the core as suggested by equations (14.91–2) and (14.91–3) is not done.