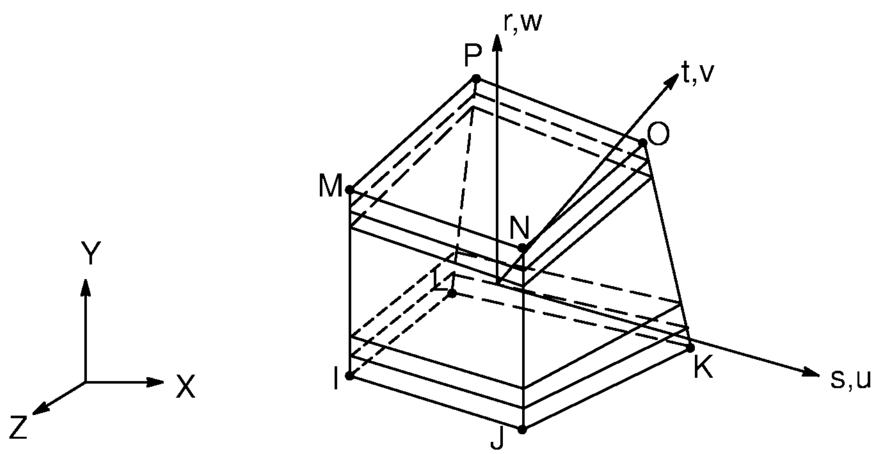
14.46 SOLID46 — 3–D Layered Structural Solid



<table>
<tr><th>Matrix or Vector</th><th colspan="2">Shape Functions</th><th>Integration Points</th></tr>
<tr><td>Stiffness Matrix</td><td colspan="2">Equations (12.8.18–1), (12.8.18–2), and (12.8.18–3) and, if modified extra shape functions are included (KEYOPT(1)≠1) and element has 8 unique nodes (12.8.19–1), (12.8.19–2), and (12.8.19–3)</td><td>2 x 2 x 2</td></tr>
<tr><td>Mass Matrix</td><td colspan="2">Equations (12.8.18–1), (12.8.18–2), and (12.8.18–3)</td><td>2 x 2 x 2</td></tr>
<tr><td>Stress Stiffness Matrix</td><td colspan="2">Same as mass matrix</td><td>2 x 2 x 2</td></tr>
<tr><td>Thermal Load Vector</td><td colspan="2">Same as stiffness matrix</td><td>2 x 2 x 2</td></tr>
<tr><td rowspan="2">Pressure Load Vector</td><td>Quad</td><td>Equation (12.5.8–1) and (12.5.8–2)</td><td>2 x 2</td></tr>
<tr><td>Triangle</td><td>Equation (12.5.1–1) and (12.5.1–2)</td><td>3</td></tr>
</table>

Load Type	Distribution
Element Temperature	Trilinear thru element
Nodal Temperature	Trilinear thru element
Pressure	Bilinear across each face

References: Wilson(38), Taylor et al(49)

14.46.1 Other Applicable Sections

Chapter 2 describes the derivation of structural element matrices and load vectors as well as stress evaluations. Section 13.1 describes integration point locations. The theory of SOLID46 is analogous to that given with SHELL99 (Section 14.99), except as given in this section.

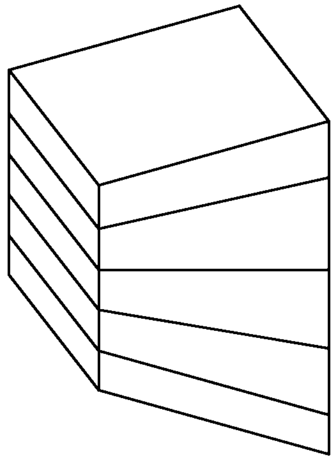
14.46.2 Assumptions and Restrictions

All material orientations are assumed to be parallel to the reference plane, even if the element has nodes inferring warped layers.

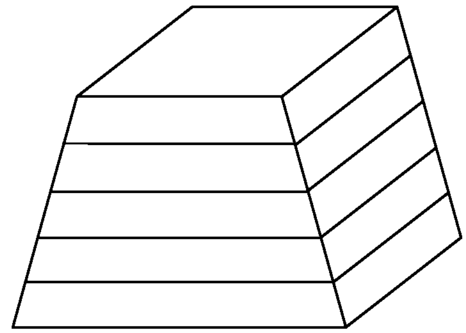
The numerical integration scheme for the thru–thickness effects are identical to that used in SHELL99. This may yield a slight numerical inaccuracy for elements having a significant change of size of layer area in the thru–thickness direction (see Figure 14.46–1). The main reason for such discrepancy stems from the approximation of the variation of the determinant of the Jacobian in the thru–thickness direction. The error is usually insignificant. However, users may want to try a patch–test problem to assess accuracy for their particular circumstances.

Unlike shell elements, SOLID46 cannot assume a zero transverse shear stiffness at the top and bottom surfaces of the element. Hence the interlaminar shear stress must be computed without using this assumption, which leads to relatively constant values thru the element.

The use of effective (“eff”) material properties developed below is based on heuristic arguments and numerical experiences rather than on a rigorous theoretical formulation. The fundamental difficulty is that multilinear displacement fields are attempted to be modeled by a linear (or perhaps quadratic) displacement shape function since the number of DOF per element must be kept to a minimum. A more rigorous solution can always be obtained by using more elements in the thru–the–layer direction. Numerical experimentation across a variety of problems indicates that the techniques used with SOLID46 give reasonable answers in most cases.



Parallel Faces at End of Layers (Thru–thickness integration close to exact)



Non–parallel Faces at End of Layers (Thru–thickness integration with slight inaccuracy)

Figure 14.46–1 Offset Geometry

14.46.3 Stress–Strain Relationships

For layer j, the stress–strain relationships in the layer coordinate system are (from equations (2.1–12) thru (2.1–17):

$$
\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} = \begin{Bmatrix} \alpha_{x,j}\Delta T \\ \alpha_{y,j}\Delta T \\ \alpha_{z,j}\Delta T \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} \frac{1}{E_{x,j}} & \frac{-\nu_{xy,j}}{E_{y,j}} & \frac{-\nu_{xz,j}}{E_{z,j}} & 0 & 0 & 0 \\ \frac{-\nu_{xy,j}}{E_{y,j}} & \frac{1}{E_{y,j}} & \frac{-\nu_{yz,j}}{E_{z,j}} & 0 & 0 & 0 \\ \frac{-\nu_{xz,j}}{E_{z,j}} & \frac{-\nu_{yz,j}}{E_{z,j}} & \frac{1}{E_{z,j}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy,j}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz,j}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xz,j}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} \quad (14.46\text{–}1)
$$

where:

$\alpha_{x,j}$ = coefficient of thermal expansion of layer j in the layer x–direction (input as ALPX on **MP** command)

$E_{x,j}$ = Young's modulus of layer j in the layer x–direction (input as EX on **MP** command)

$G_{xy,j}$ = shear modulus of layer j in layer x–y plane (input as GXY on **MP** command)

$\nu_{xy,j}$ = Poisson's ratio of layer j in x–y plane (input as NUXY on **MP** command)

ΔT = $T - T_{ref}$

T = temperature at point being studied

T_{ref} = reference temperature (input on **TREF** command)

To help ensure continuity of stresses between the layers, equation (14.46–1) is modified to yield:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} = \begin{Bmatrix} \alpha_{x,j}\Delta T \\ \alpha_{y,j}\Delta T \\ \alpha_z^{eff}\Delta T \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{bmatrix} \frac{1}{E_{x,j}} & \frac{-\nu_{xy,j}}{E_{y,j}} & \frac{-\nu_{xz,j}^{eff}}{E_z^{eff}} & 0 & 0 & 0 \\ \frac{-\nu_{xy,j}}{E_{y,j}} & \frac{1}{E_{y,j}} & \frac{-\nu_{yz,j}^{eff}}{E_z^{eff}} & 0 & 0 & 0 \\ \frac{-\nu_{xz,j}^{eff}}{E_z^{eff}} & \frac{-\nu_{yz,j}^{eff}}{E_z^{eff}} & \frac{1}{E_z^{eff}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy,j}} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{11,j}^G & D_{21,j}^G \\ 0 & 0 & 0 & 0 & D_{12,j}^G & D_{22,j}^G \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} \tag{14.46–2}$$

where:

$$\alpha_z^{eff} = \frac{\sum_{j=1}^{N\ell} t_j \ \alpha_{z,j}}{t_{TOT}}$$

where:

$$\nu_{xz,j}^{eff} = \begin{cases} C & \text{if } C < .45 \\ \text{or} & \\ \nu_{xz,j} & \text{if } C \geq .45 \end{cases}$$

$$C = \nu_{xz,j} \frac{E_z^{eff}}{E_{z,j}}$$

$$E_z^{eff} = \frac{t_{TOT}}{\sum_{j=1}^{N\ell} \frac{t_j}{E_{z,j}}}$$

$$[D^G]_j = \begin{bmatrix} D_{11,j}^G & D_{21,j}^G \\ D_{12,j}^G & D_{22,j}^G \end{bmatrix} = \left([T]_j^{-1}\right)^T \left[d^G\right] [T]_j^{-1} =$$ effective inverted shear moduli in layer system

$[d^G]$ = $\frac{1}{t_{TOT}} \sum_{j=1}^{N_\ell} t_j \left[A_\ell\right]_j^{-1}$ = effective inverted shear moduli in element system

$\left[A_\ell\right]_j$ = $[T]_j^T [D_z]_j [T]_j$

$[T]_j$ = transformation matrix to convert from layer to element systems

$[D_z]_j$ = $\begin{bmatrix} G_{yz,j} & 0 \\ 0 & G_{xz,j} \end{bmatrix}$

t_j = average thickness of layer j

t_{TOT} = average total thickness of element

N_ℓ = number of layers

14.46.4 General Strain and Stress Calculations

The following steps are used to compute strains and stresses at a typical point within layer j:

1. The strain vector ($\lfloor \epsilon_x,\ \epsilon_y,\ \epsilon_z,\ \epsilon_{xy},\ \epsilon_{yz},\ \epsilon_{xz} \rfloor$) is determined from the displacements and the strain–displacement relationships, evaluated at the point of interest.
2. The strains are converted from element to layer coordinates.
3. The strains are adjusted for thermal effects, with the effective coefficient of thermal expansion in the z–direction being:

$$\alpha_z^{eff} = \frac{\sum_{j=1}^{N_\ell} t_j\ \alpha_{z,j}}{t_{TOT}} \tag{14.46–3}$$

4. The normal strain is then adjusted with

$$\epsilon_z' = \epsilon_z \frac{E_z^{eff}}{E_{z,j}} \tag{14.46–4}$$

5. The transverse shear strains are computed by way of the stresses all in the layer coordinate system.

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \left[D^G\right]^{-1} \begin{Bmatrix} \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} \tag{14.46–5}$$

where: $\varepsilon_{yz}, \varepsilon_{xz}$ = shear strains based on strain–displacement relationships

Then,

$$\epsilon'_{yz,j} = \sigma_{yz} \;/\; G_{yz,j} \tag{14.46–6}$$

$$\epsilon'_{xz,j} = \sigma_{xz} \;/\; G_{xz,j} \tag{14.46–7}$$

where: $\epsilon'_{yz}, \epsilon'_{xz}$=shear strains based on continuity of shearing stresses

6. Finally, the strains are converted to stresses by the usual relationship:

$$\{\sigma\}_j = [D]_j \left(\{\epsilon\}_j - \{\epsilon^{th}\}_j \right) \tag{14.46–8}$$

where: $[D]_j$ = inverse of stress–strain matrix used in equation (14.46–1)

7. If the element has more than one layer and any layer has $\nu^{eff}_{xz,j}$ or $\nu^{eff}_{yz,j}$ exceeding 0.45, the normal stresses are computed based on nodal forces.

14.46.5 Interlaminar Shear Stress Calculation

It may be seen that the interlaminar shear stress will, in general, not be zero at a free surface. This is because the element has no knowledge as to whether or not the top or bottom face is a free surface or if there is another element attached to that face.

There are two methods of calculating interlaminar shear stress: by nodal forces and by evaluating stresses layer–by–layer.

Nodal Forces

The shear stresses over the entire volume are computed to be:

$$\sigma_{xz} = \frac{1}{4}\left[\frac{F^x_M - F^x_I}{A^{I-M}} + \frac{F^x_N - F^x_J}{A^{J-N}} + \frac{F^x_O - F^x_K}{A^{K-O}} + \frac{F^x_P - F^x_L}{A^{L-P}} \right] \tag{14.46–9}$$

$$\sigma_{yz} = \frac{1}{4}\left[\frac{F^y_M - F^y_I}{A^{I-M}} + \frac{F^y_N - F^y_J}{A^{J-N}} + \frac{F^y_O - F^y_K}{A^{K-O}} + \frac{F^y_P - F^y_L}{A^{L-P}} \right] \tag{14.46–10}$$

where: σ_{xz}, σ_{yz} = output quantities AVERAGE TRANSVERSE SHEAR STRESS COMPONENTS

F_I^x, F_I^y, etc. = forces at node I (etc.) parallel to the reference plane, with x being parallel to the element x direction

A^{I-M}, etc. = tributary area for node (evaluated by using the determinant of the Jacobian at the nearest integration point in base plane)

Evaluating Stresses Layer–by–Layer

This option is available only if KEYOPT(2) = 0 or 1 and simply uses the layer shear stresses for the interlaminar stresses. Thus, the interlaminar shear stresses in the element x direction are:

$$\sigma_{xz}^{1} = \sigma_{xz} \text{ at bottom of layer 1 (in plane I–J–K–L)} \tag{14.46–11}$$

$$\sigma_{xz}^{N\ell+1} = \sigma_{xz} \text{ at top of layer NL (in plane M–N–O–P)} \tag{14.46–12}$$

$$\sigma_{xz}^{j} = \frac{1}{2}\,(\sigma_{xz} \text{ at top of layer j–1} + s_{xz} \text{ at bottom of layer j}) \quad \text{where } 1 < j < N_\ell \tag{14.46–13}$$

The σ_{xz} terms are the shear stresses computed from equation (14.46–8), except that the stresses have been converted to element coordinates. The interlaminar shear stresses in the element y direction are analogous. The maximum of the vector sum of σ_{xz} and σ_{yz} for all layers is printed with the label “MAXIMUM INTERLAMINAR SHEAR STRESS”.