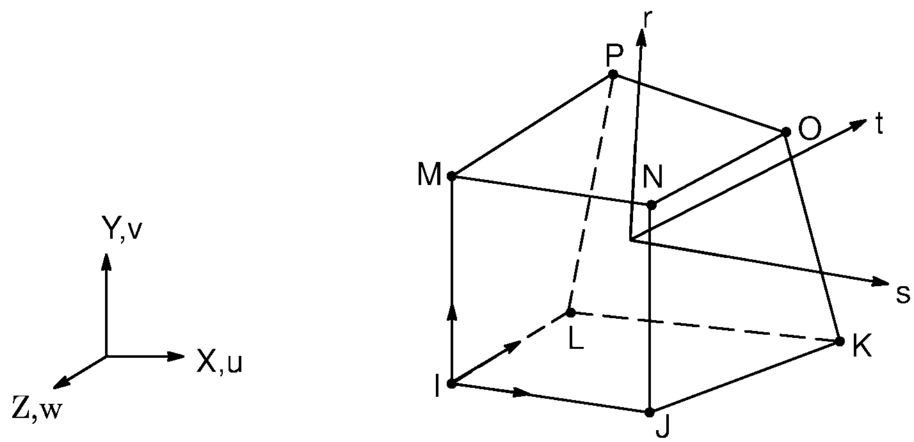
14.65 SOLID65 — 3–D Reinforced Concrete Solid



Matrix or Vector	Shape Functions		Integration Points
Stiffness Matrix	Equations (12.8.18–1), (12.8.18–2) and (12.8.18–3), or if modified extra shape functions are included (KEYOPT(1)=0) and element has 8 unique nodes equations (12.8.19–1), (12.8.19–2) and (12.8.19–3)		2 x 2 x 2
Mass Matrix	Equations (12.8.18–1), (12.8.18–2) and (12.8.18–3)		2 x 2 x 2
Thermal Load Vector	Same as stiffness matrix		2 x 2 x 2
Pressure Load Vector	Quad	Equation (12.5.8–1) and (12.5.8–2)	2 x 2
	Triangle	Equation (12.5.1–1) and (12.5.1–2)	3

Load Type	Distribution
Element Temperature	Trilinear thru element
Nodal Temperature	Trilinear thru element
Pressure	Bilinear across each face

References: William and Warnke(37), Wilson(38), Taylor(49)

14.65.1 Assumptions and Restrictions

1. Cracking is permitted in three orthogonal directions at each integration point.
2. If cracking occurs at an integration point, the cracking is modeled through an adjustment of material properties which effectively treats the cracking as a "smeared band" of cracks, rather than discrete cracks.
3. The concrete material is assumed to be initially isotropic.
4. Whenever the reinforcement capability of the element is used, the reinforcement is assumed to be "smeared" throughout the element.
5. In addition to cracking and crushing, the concrete may also undergo plasticity, with the Drucker–Prager failure surface being most commonly used. In this case, the plasticity is done before the cracking and crushing checks.

14.65.2 Description

SOLID65 allows the presence of four different materials within each element; one matrix material (e.g. concrete) and a maximum of three independent reinforcing materials. The concrete material is capable of directional integration point cracking and crushing besides incorporating plastic and creep behavior. The reinforcement (which also incorporates creep and plasticity) has uniaxial stiffness only and is assumed to be smeared throughout the element. Directional orientation is accomplished through user specified angles.

14.65.3 Linear Behavior – General

The stress–strain matrix [D] used for this element is defined as:

$$[D] = \left(1 - \sum_{i=1}^{N_r} V_i^R\right) [D^c] + \sum_{i=1}^{N_r} V_i^R \, [D^r]_i \qquad (14.65\text{–}1)$$

where: N_r = number of reinforcing materials (maximum of three, all reinforcement is ignored if MAT1 (explained below) equals zero. Also, if MAT1, MAT2, or MAT3 equals the concrete material number, the reinforcement with that material number is ignored)

V_i^R = ratio of the volume of reinforcing material i to the total volume of the element (input as VRi on **R** command)

$[D^c]$ = stress–strain matrix for concrete, defined by equation (14.65–2)

$[D^r]_i$ = stress–strain matrix for reinforcement i, defined by equation (14.65–3)

MAT1, MAT2, MAT3=material numbers associated with material behavior of reinforcement (input as MAT1, MAT2, and MAT3 on **R** command)

14.65.4 Linear Behavior – Concrete

The matrix $[D^c]$ is derived by specializing the orthotropic stress–strain relations defined by equation (2.1–4) to the case of an isotropic material or

$$[D^c] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \quad (14.65\text{–}2)$$

where: E = Young's modulus for concrete (input as EX on **MP** command)

ν = Poisson's ratio for concrete (input as PRXY or NUXY on **MP** command)

14.65.5 Linear Behavior – Reinforcement

The orientation of the reinforcement i within an element is depicted in Figure 14.65–1. The element coordinate system is denoted by (X, Y, Z) and $(x_i^r,\ y_i^r,\ z_i^r)$ describes the coordinate system for reinforcement type i. The stress–strain matrix with respect to each coordinate system $(x_i^r,\ y_i^r,\ z_i^r)$ has the form

$$\begin{Bmatrix} \sigma^r_{xx} \\ \sigma^r_{yy} \\ \sigma^r_{zz} \\ \sigma^r_{xy} \\ \sigma^r_{yz} \\ \sigma^r_{xz} \end{Bmatrix} = \begin{bmatrix} E^r_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon^r_{xx} \\ \epsilon^r_{yy} \\ \epsilon^r_{zz} \\ \epsilon^r_{xy} \\ \epsilon^r_{yz} \\ \epsilon^r_{xz} \end{Bmatrix} = \left[D^r\right]_i \begin{Bmatrix} \epsilon^r_{xx} \\ \epsilon^r_{yy} \\ \epsilon^r_{zz} \\ \epsilon^r_{xy} \\ \epsilon^r_{yz} \\ \epsilon^r_{xz} \end{Bmatrix} \qquad (14.65\text{–}3)$$

where: E^r_i = Young's modulus of reinforcement type i (input as EX on **MP** command)

It may be seen that the only nonzero stress component is σ^r_{xx}, the axial stress in the x^r_i direction of reinforcement type i. The reinforcement direction x^r_i is related to element coordinates X, Y, Z through

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} \cos\theta_i\ \cos\phi_i \\ \sin\theta_i\ \cos\phi_i \\ \sin\phi_i \end{Bmatrix} x^r_i = \begin{Bmatrix} \ell^r_1 \\ \ell^r_2 \\ \ell^r_3 \end{Bmatrix} x^r_i \qquad (14.65\text{–}4)$$

where: θ_i = angle between the projection of the x^r_i axis on XY plane and the X axis (input quantities THETA1, THETA2, and THETA3 on **R** command)

ϕ_i = angle between the x^r_i axis and the XY plane (input quantities PHI1, PHI2, and PHI3 on **R** command)

ℓ^r_i = direction cosines between x^r_i axis and element X, Y, Z axes

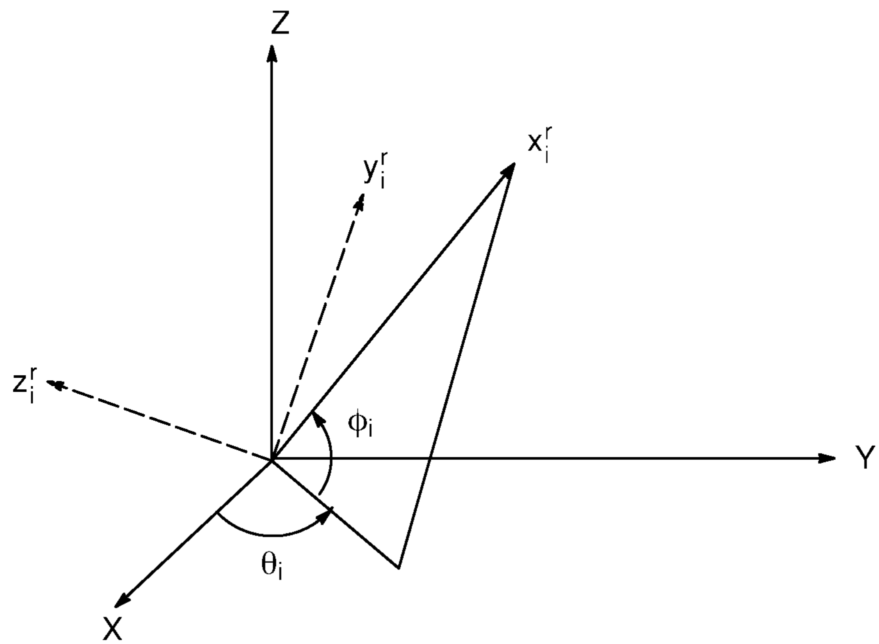


Figure 14.65–1 Reinforcement Orientation

Since the reinforcement material matrix is defined in coordinates aligned in the direction of reinforcement orientation, it is necessary to construct a transformation of the form

$$\left[D^R\right]_i = [T^r]^T[D^r]_i[T^r] \quad (14.65\text{–}5)$$

in order to express the material behavior of the reinforcement in global coordinates. The form of this transformation by Schnobrich(29) is

$$[T^r] = \begin{bmatrix} a_{11}^2 & a_{12}^2 & a_{13}^2 & a_{11}a_{12} & a_{12}a_{13} & a_{11}a_{13} \\ a_{21}^2 & a_{22}^2 & a_{23}^2 & a_{21}a_{22} & a_{22}a_{23} & a_{21}a_{23} \\ a_{31}^2 & a_{32}^2 & a_{33}^2 & a_{31}a_{32} & a_{32}a_{33} & a_{31}a_{33} \\ 2a_{11}a_{21} & 2a_{12}a_{22} & 2a_{13}a_{23} & a_{11}a_{22} + a_{12}a_{21} & a_{12}a_{23} + a_{13}a_{32} & a_{11}a_{23} + a_{13}a_{21} \\ 2a_{21}a_{31} & 2a_{22}a_{32} & 2a_{23}a_{33} & a_{21}a_{32} + a_{22}a_{31} & a_{22}a_{33} + a_{23}a_{32} & a_{21}a_{33} + a_{13}a_{21} \\ 2a_{11}a_{31} & 2a_{12}a_{32} & 2a_{13}a_{33} & a_{11}a_{32} + a_{12}a_{31} & a_{12}a_{33} + a_{13}a_{32} & a_{11}a_{33} + a_{13}a_{31} \end{bmatrix} \quad (14.65\text{–}6)$$

where the coefficients a_{ij} are defined as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \ell_1^r & \ell_2^r & \ell_3^r \\ m_1^r & m_2^r & m_3^r \\ n_1^r & n_2^r & n_3^r \end{bmatrix} \quad (14.65\text{–}7)$$

The vector $\lfloor \ell_1^r \quad \ell_2^r \quad \ell_3^r \rfloor^T$ is defined by equation (14.65–4) while $\lfloor m_1^r \quad m_2^r \quad m_3^r \rfloor^T$ and $\lfloor n_1^r \quad n_2^r \quad n_3^r \rfloor^T$ are unit vectors mutually orthogonal to $\lfloor \ell_1^r \quad \ell_2^r \quad \ell_3^r \rfloor^T$ thus defining a Cartesian coordinate referring to reinforcement directions. If the operations presented by equation (14.65–5) are performed substituting equation (14.65–3) and equation (14.65–6), the resulting reinforcement material matrix in element coordinates takes the form

$$[D^r]_i = E_i^r \, \{A_d\} \, \{A_d\}^T \quad (14.65\text{–}8)$$

where: $\{A_d\} = \lfloor a_{11}^2 \quad a_{21}^2 \; ... \; a_{11} \, a_{13} \rfloor^T$

Therefore, the only direction cosines used in $[D^R]_i$ involve the uniquely defined unit vector $\lfloor \ell_1^r \quad \ell_2^r \quad \ell_3^r \rfloor^T$.

14.65.6 Nonlinear Behavior – Concrete

As mentioned previously, the matrix material (e.g. concrete) is capable of plasticity, creep, cracking and crushing. The plasticity and creep formulations are the same as those implemented in SOLID45 (see Section 4.1). The concrete material model with its cracking and crushing capabilities is discussed in Section 4.7. This material model predicts either elastic behavior, cracking behavior or crushing behavior. If elastic behavior is predicted, the concrete is treated as a linear elastic material (discussed above). If cracking or crushing behavior is predicted, the elastic, stress–strain matrix is adjusted as discussed below for each failure mode.

14.65.7 Modeling of a Crack

The presence of a crack at an integration point is represented through modification of the stress–strain relations by introducing a plane of weakness in a direction normal to the crack face. Also, a shear transfer coefficient β_t (constant C_1 on the **TBDATA** command with **TB**,CONCR) is introduced which represents a shear strength reduction factor for those subsequent loads which induce sliding (shear) across the crack face. The stress–strain relations for a material that has cracked in one direction only become:

$$\left[D_c^{ck}\right] = \frac{E}{(1+\nu)} \begin{bmatrix} \frac{R^t(1+\nu)}{E} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ 0 & \frac{\nu}{1-\nu} & \frac{1}{1-\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_t}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2} \end{bmatrix} \qquad (14.65\text{–}9)$$

where the superscript ck signifies that the stress strain relations refer to a coordinate system parallel to principal stress directions with the x^{ck} axis perpendicular to the crack face. If KEYOPT(7) = 0, R^t = 0.0. If KEYOPT(7) = 1, R^t is the slope (secant modulus) as defined in Figure 14.65–2. R^t works with adaptive descent and diminishes to 0.0 as the solution converges

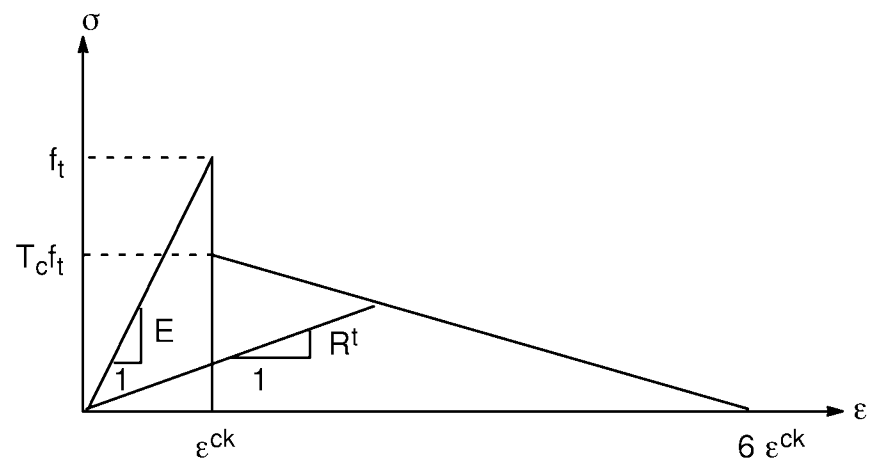


Figure 14.65–2 Strength of Cracked Condition

In Figure 14.65–2,

f_t = unaxial tensile cracking stress (input as C_3 on the **TBDATA** command)

T_c = multiplier for amount of tensile stress relaxation (input as C_9 on the **TBDATA** command, defaults to 0.6)

If the crack closes, then all compressive stresses normal to the crack plane are transmitted across the crack and only a shear transfer coefficient β_c (constant C_2 on the

TBDATA command with **TB**,CONCR) for a closed crack is introduced. Then $\left[D_c^{ck}\right]$ can be expressed as

$$\left[D_c^{ck}\right] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_c\frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_c\frac{(1-2\nu)}{2} \end{bmatrix} \quad (14.65\text{–}10)$$

The stress–strain relations for concrete that has cracked in two directions are:

$$\left[D_c^{ck}\right] = E\begin{bmatrix} \frac{R^t}{E} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R^t}{E} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} \end{bmatrix} \quad (14.65\text{–}11)$$

If both directions reclose,

$$\left[D_c^{ck}\right] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_c\frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_c\frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_c\frac{(1-2\nu)}{2} \end{bmatrix} \qquad (14.65–12)$$

The stress–strain relations for concrete that has cracked in all three directions are:

$$\left[D_c^{ck}\right] = E\begin{bmatrix} \frac{R^t}{E} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R^t}{E} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{R^t}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta_t}{2(1+\nu)} \end{bmatrix} \qquad (14.65–13)$$

If all three cracks reclose, equation (14.65–12) is followed. In total there are 16 possible combinations of crack arrangement and appropriate changes in stress–strain relationships incorporated in SOLID65. A note is output if $1 > \beta_c > \beta_t > 0$ are not true.

The transformation of $\left[D_c^{ck}\right]$ to element coordinates has the form

$$[D_c] = \left[T^{ck}\right]^T\left[D_c^{ck}\right]\left[T^{ck}\right] \qquad (14.65–14)$$

where $[T^{ck}]$ has a form identical to equation (14.65–6) and the three columns of [A] in equation (14.65–7) are now the principal direction vectors.

The open or closed status of integration point cracking is based on a strain value ϵ_{ck}^{ck} called the crack strain. For the case of a possible crack in the x direction, this strain is evaluated as

$$\epsilon_{ck}^{ck} = \begin{cases} \epsilon_x^{ck} + \dfrac{\nu}{1-\nu}(\epsilon_y^{ck} + \epsilon_z^{ck}) & \text{if no cracking has occurred} \\ \epsilon_x^{ck} + \nu\,\epsilon_z^{ck} & \text{if y direction has cracked} \\ \epsilon_x^{ck} & \text{if y and z direction have cracked} \end{cases} \tag{14.65–15}$$

where:

ϵ_x^{ck}, ϵ_y^{ck}, and ϵ_z^{ck} = three normal component strains in crack orientation.

The vector $\{\varepsilon^{ck}\}$ is computed by:

$$\left\{\epsilon^{ck}\right\} = \left[T^{ck}\right]\left\{\epsilon'\right\} \tag{14.65–16}$$

where: $\{\varepsilon'\}$ = modified total strain (in element coordinates)

$\{\varepsilon'\}$, in turn, is defined as:

$$\left\{\epsilon_n'\right\} = \left\{\epsilon_{n-1}^{el}\right\} + \{\Delta\epsilon_n\} - \left\{\Delta\epsilon_n^{th}\right\} - \left\{\Delta\epsilon_n^{pl}\right\} \tag{14.65–17}$$

where:
- n = substep number
- $\left\{\epsilon_{n-1}^{el}\right\}$ = elastic strain from previous substep
- $\{\Delta\varepsilon_n\}$ = total strain increment (based on $\{\Delta u_n\}$, the displacement increment over the substep)
- $\left\{\Delta\epsilon_n^{th}\right\}$ = thermal strain increment
- $\left\{\Delta\epsilon_n^{pl}\right\}$ = plastic strain increment

If ϵ_{ck}^{ck} is less than zero, the associated crack is assumed to be closed.

If ϵ_{ck}^{ck} is greater than or equal to zero, the associated crack is assumed to be open. When cracking first occurs at an integration point, the crack is assumed to be open for the next iteration.

14.65.8 Modeling of Crushing

If the material at an integration point fails in uniaxial, biaxial, or triaxial compression, the material is assumed to crush at that point. In SOLID65, crushing is defined as the complete deterioration of the structural integrity of the material (e.g. material spalling).

Under conditions where crushing has occurred, material strength is assumed to have degraded to an extent such that the contribution to the stiffness of an element at the integration point in question can be ignored.

14.65.9 Nonlinear Behavior – Reinforcement

The one–dimensional creep and plasticity behavior for SOLID65 reinforcement is modeled in the same manner as for LINK8.